45. Some Trace Relations of Twisting Operators on the Spaces of Cusp Forms of Half-integral Weight

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In the papers [3] and [4], we calculated the traces of Hecke operators $\tilde{T}(n^2)$ on the space of cusp forms of half-integral weight $S(k+1/2, N, \chi)$ and on the Kohnen subspace $S(k+1/2, N, \chi)_K$. Moreover we found that the above traces are linear combinations of the traces of certain operators on the spaces $S(2k, N')$ (N' runs over divisors of $N/2$). In this paper, we report similar trace relations of the twisting operators on the spaces $S(k+1/2, N, \chi)$ and $S(k+1/2, N, \chi)_K$. Details will appear in [5].

Preliminaries. (a) **General notations.** Let k denote a positive integer. If $z \in \boldsymbol{C}$ and $x \in \boldsymbol{C}$, we put $z^x = \exp(x \cdot \log(z))$ with $\log(z) = \log(|z|) + \sqrt{-1}\arg(z)$, $\arg(z)$ being determined by $-\pi < \arg(z) \leq \pi$. Also we put $\boldsymbol{e}(z) = \exp(2\pi\sqrt{-1}z)$.

Let $\mathfrak{H}$ be the complex upper half plane. For a complex-valued function $f(z)$ on $\mathfrak{H}$, $\alpha = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2^+(\boldsymbol{R})$, $\gamma = \begin{pmatrix} u & v \\ w & x \end{pmatrix} \in \Gamma_0(4)$ and $z \in \mathfrak{H}$, we define functions $J(\alpha, z)$, $j(\gamma, z)$ and $f|[\alpha]_k(z)$ on $\mathfrak{H}$ by: $J(\alpha, z) = cz + d$, $j(\gamma, z) = \left(\frac{-1}{x}\right)^{-1/2} \left(\frac{w}{x}\right)(wz+x)^{1/2}$ and $f|[\alpha]_k(z) = (\det \alpha)^{k/2} J(\alpha, z)^{-k} f(\alpha z)$.

For a real number x, $[x]$ means the greatest integer m with $x \geq m$. $|\ |_p$ is the p-adic absolute value which is normalized with $|p|_p = p^{-1}$. See [1, p. 82] for the definition of the Kronecker symbol $\left(\frac{a}{b}\right)$ (a, b integers with $(a, b) \neq (0, 0)$). Let N be a positive integer and m an integer $\neq 0$. We write $m | N^\infty$ if every prime factor of m divides N. For a finite-dimensional vector space V over $\boldsymbol{C}$ and a linear operator T on V, $\mathrm{tr}(T|V)$ denotes the trace of T on V.

(b) **Modular forms of integral weight.** Let N be a positive integer. By $S(2k, N)$, we denote the space of all holomorphic cusp forms of weight $2k$ with the trivial character on the group $\Gamma = \Gamma_0(N)$.

Let $\alpha \in GL_2^+(\boldsymbol{R})$. If Γ and $\alpha^{-1}\Gamma\alpha$ are commensurable, we define a linear operator $[\Gamma\alpha\Gamma]_{2k}$ on $S(2k, N)$ by: $f|[\Gamma\alpha\Gamma]_{2k} = (\det \alpha)^{k-1} \sum_{\alpha_i} f|[\alpha_i]_{2k}$, where α_i runs over a system of representatives for $\Gamma \backslash \Gamma\alpha\Gamma$. For a natural number n with $(n, N) = 1$, we put $T(n) = T_{2k,N}(n) = \sum_{ad=n} \left[\Gamma \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \Gamma\right]_{2k}$, where the sum is extended over all pairs of integers (a, d) such that $a, d > 0$, $a | d$, $ad = n$. Moreover let Q be a positive divisor of N such that $(Q, N/Q) = 1$ and $Q \neq 1$.