44. On 3-Connected 10-Dimensional Manifolds*)

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1. Introduction. In [4], the author gave a complete homotopy classification of 2-connected smooth 8-manifolds with vanishing 4th homology groups as an application of the homotopy classification theory of primary manifolds. In this paper, as another application of it, we classify 3-connected smooth 10-manifolds M satisfying the following hypotheses:

(H1) $H_4(M)$ is torsion free.

(H2) The tangent bundle of M is trivial on its 4-skeleton.

It is easily seen that (H2) is equivalent to

(H2′) The tangent bundle of M is stably trivial.

Thus, our classification is that of 3-connected 10-dimensional π-manifolds with torsion free homology groups. Henthforth, manifolds are smooth, oriented, connected, and closed unless mentioned explicitly. Homotopy equivalences and diffeomorphisms are orientation preserving. The proofs of the theorems are given briefly.

Theorem 1. *Let M be a 3-connected 10-manifold satisfying (H1), (H2). Then, there exists a connected sum decomposition*

$$M=M_1\#(S^5\times S^5)\#\cdots\#(S^5\times S^5),$$

where M_1 is a 3-connected 10-manifold satisfying (H1), (H2) and $H_5(M_1)=0$. The decomposition is unique up to diffeomorphism, that is, if there exists another decomposition as above by M_1' and $S^5\times S^5$'s, then M_1, M_1' must be diffeomorphic and the numbers of $S^5\times S^5$ are equal.

Let $\mathcal{H}(p+q+1, r, q)$ be the set of the handlebodies obtained by gluing q-handles, r in number, to a $(p+q+1)$-disk. In the following theorem, the symmetric bilinear form $\psi: H^4(M)\times H^4(M)\to Z_2$ is defined by $\psi(x,y)=\langle S_q^2x_2\cup y_2, [M]_2\rangle$, where x_2, y_2, $[M_2]$ denote x, y, $[M]$ in Z_2 coefficient respectively. The type of M is defined by using ψ (cf. [2], [3]). So, it is a homotopy invariant of M and coincides with the type of the handlebody W of $\mathcal{H}(11, r, 6)$, $r=\operatorname{rank} H_4(M)$, bounded by M up to homotopy equivalence (cf. Theorem 8.3 of [2]). The following theorem completes our classification up to diffeomorphism mod Θ_{10}.

Theorem 2. *Let M, M' be 3-connected 10-manifolds satisfying the hypothesis (H2) and $H_5(M)=H_5(M')=0$ (so, (H1) is satisfied). Then, M, M' are diffeomorpic mod Θ_{10} if and only if M, M' are homotopy equivalent. Such manifolds M with fixed $\operatorname{rank} H_4(M)=r$ can be completely classified up to homotopy equivalence, and hence up to diffeomorphism mod Θ_{10}, and the*

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