43. *q*-analogue of de Rham Cohomology Associated with Jackson Integrals. I

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In this note we want to give a new formulation of Jackson integrals involved in basic hypergeometric functions through the classical Barnes' representations. We define a q-analogue of de Rham cohomology which can be formulated by means of q-version of Sato's b-functions and derive associated holonomic q-difference system. The evaluation of its multiplicity will be given as a number of different asymptotics.

1. *Structure of b-functions.* We take the elliptic modulus $q=e^{2\pi i\tau}$, $\mathrm{Im}\,\tau>0$. Let X be an n dimensional integer lattice $\simeq \boldsymbol{Z}^n$. We put $\bar{X}=X\otimes \boldsymbol{C}^*$, the n dimensional algebraic torus twisted by q. Let $\chi_1, \chi_2, \cdots, \chi_n$ be a basis of X such that an arbitrary $\chi\in X$ can be uniquely written by $\chi=\sum_{j=1}^n \nu_j\chi_j$, $\nu_j\in \boldsymbol{Z}$. We may identify $\bar{X}$ isomorphic to $X\otimes(\boldsymbol{C}/(2\pi i/\log q))$ with the direct product of n pieces of $\boldsymbol{C}^*$. The inclusion $X\subset\bar{X}$ can be obtained by identifying χ_j with the element $t=(1,\cdots,1,q,1,\cdots,1)\in(\boldsymbol{C}^*)^n$. We denote by Q_j the shift operator $Q_jf(t)=f(\chi_j\cdot t)$ induced by the displacement $t\to\chi_j\cdot t$ for a function f on $\bar{X}$. We put $Q^\chi=Q_1^{\nu_1}\cdots Q_n^{\nu_n}$. We consider the q-difference equations

$$Q^\chi\Phi(t)=b_\chi(t)\Phi(t),\qquad \chi\in X \text{ and } t\in\bar{X}, \tag{1.1}$$

for a set of rational functions $\{b_\chi(t)\}_{\chi\in X}$, on $\bar{X}$, which are not identically zero. $\{b_\chi(t)\}_{\chi\in X}$ satisfies the compatibility condition

$$b_{\chi+\chi'}(t)=b_\chi(t)\cdot Q^\chi b_{\chi'}(t), \tag{1.2}$$

so that $\{b_\chi(t)\}_{\chi\in X}$ defines a 1-cocycle on X with values in $R^\times(\bar{X})$ the multiplicative abelian group consisting of non-zero rational functions on $\bar{X}$. We denote by $R(\bar{X})$ the field of rational functions on $\bar{X}$. $\{b_\chi(t)\}_{\chi\in X}$ is a coboundary if and only if $b_\chi(t)=Q^\chi\varphi(t)/\varphi(t)$ for $\varphi\in R^\times(\bar{X})$. We write the corresponding 1-cohomology by $H^1(X, R^\times(\bar{X}))$.

We put $(x)_\infty=\prod_{\nu=0}^\infty(1-xq^\nu)$ and $(x)_n=(x)_\infty/(xq^n)_\infty$ for $n\in\boldsymbol{Z}$. Then the following important result holds.

Proposition. *An arbitrary cocycle* $\{b_\chi(t)\}_{\chi\in X}$ *modulo a coboundary can be expressed by* (1.1), *where* Φ *denotes a q-multiplicative function on* $\bar{X}$ *written by*

$$\Phi=\prod_{j=1}^n t_j^{\alpha_j}\prod_{j=1}^m\frac{(a_j't^{\mu_j})_\infty}{(a_jt^{\mu_j})_\infty} \tag{1.3}$$

for some non-negative integer m and α_j, a_j', $a_j\in\boldsymbol{C}$, *and for* $\mu_j\in\check{X}=\mathrm{Hom}(X,\boldsymbol{Z})$. t^{μ_j} *denotes a monomial* $t_1^{\mu_j(\chi_1)}\cdots t_n^{\mu_j(\chi_n)}$. a_j *or* a_j' *may vanish or may not.*

This is a q-version of Sato's theorem in [6] and can be proved in a