5. *Some Aspects in the Theory of Representations of Discrete Groups. II*

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Here we concern mainly with equivalence relations among irreducible unitary representations (=IURs) of an infinite wreath product group, constructed in the first part [1] of these notes. We keep to the notations in [1].

1. Commutativity of two kinds of inducing processes. Let T be a group and S its subgroup. Consider wreath product groups $\mathfrak{S}_A(S)$ and $\mathfrak{S}_A(T)$. Then we have two kinds of inducing of representations: the usual one and the WP-inducing. We give a certain commutativity of these inducing processes. Start with a datum $R=\{A, \rho_S, \chi, a=(a_\alpha)_{\alpha\in A}\}$ for an elementary representation of $\rho(R)$ of $\mathfrak{S}_A(S)$. On the one hand, put $\tilde{\rho}_T=\mathrm{Ind}_S^T \rho_S$, and let $\tilde{a}_\alpha=\mathrm{Ind}_S^T a_\alpha \in V(\tilde{\rho}_T)$ be the induced vector of $a_\alpha \in V(\rho_S)$. Then $\tilde{a}=(\tilde{a}_\alpha)_{\alpha\in A}$ is a reference vector for $(\tilde{V}_\alpha)_{\alpha\in A}$ with $\tilde{V}_\alpha=V(\tilde{\rho}_T)$, and denote it as $\tilde{a}=\mathrm{Ind}_S^T a$. Thus we get a datum $\tilde{R}=\{A, \tilde{\rho}_T, \chi, \tilde{a}\}$ for $\mathfrak{S}_A(T)$ and correspondingly an elementary representation $\rho(\tilde{R})$ of $\mathfrak{S}_A(T)$. On the other hand, we have the induced representation $\mathrm{Ind}(\rho(R); \mathfrak{S}_A(S)\uparrow\mathfrak{S}_A(T))$.

Theorem 1. *Let R be a datum for an elementary representation of $\mathfrak{S}_A(S)$. Then the two representations $\rho(\tilde{R})$ and $\mathrm{Ind}(\rho(R); \mathfrak{S}_A(S)\uparrow\mathfrak{S}_A(T))$ of $\mathfrak{S}_A(T)$ are canonically equivalent to each other. A similar assertion holds for standard representation for $\mathfrak{S}_A(S)$ and $\mathfrak{S}_A(T)$.*

2. Equivalence relations among standard representations. Take two induced representations $\rho(Q_i)=\mathrm{Ind}(\pi(Q_i); H(Q_i)\uparrow\mathfrak{S}_A(T))$, $i=1, 2$, of $\mathfrak{S}_A(T)$, called standard, and let the corresponding data be

$$Q_1=\{(A_\gamma, \rho_{T_{1\gamma}}^\gamma, \chi_{1\gamma})_{\gamma\in\Gamma}, (a_1(\gamma))_{\gamma\in\Gamma}, (b_{1\gamma})_{\gamma\in\Gamma}\},$$
$$Q_2=\{(B_\delta, \rho_{T_{2\delta}}^\delta, \chi_{2\delta})_{\delta\in\Delta}, (a_2(\delta))_{\delta\in\Delta}, (b_{2\delta})_{\delta\in\Delta}\},$$

where, in particular, $(A_\gamma)_{\gamma\in\Gamma}$ and $(B_\delta)_{\delta\in\Delta}$ are partitions of A, and $T_{1\gamma}$ and $T_{2\delta}$ are subgroups of T. For an element ζ of $\mathfrak{S}_A$, we call an *adjustment* of Q_2 by ζ the datum

$${}^\zeta Q_2=\{(\zeta(B_\delta), \rho_{T_{2\delta}}^\delta, \chi_\delta)_{\delta\in\Delta}, (a_2(\delta))_{\delta\in\Delta}, (b_{2\delta})_{\delta\in\Delta}\}.$$

Then $\rho(Q_2)$ is equivalent to $\rho({}^\zeta Q_2)$ in a trivial fashion.

Theorem 2. *Assume that two data Q_1 and Q_2 satisfy the condition (Q1), i.e., $|\Gamma_f|\leqq 1$, $|\Delta_f|\leqq 1$, and that both $\rho(Q_1)$ and $\rho(Q_2)$ are irreducible. Then they are mutually equivalent if and only if the following conditions hold.*

(EQU1) *Replacing Q_2 by its adjustment by an element in $\mathfrak{S}_A$ if necessary, we have a 1–1 correspondence κ of Γ onto Δ such that $A_\gamma=B_{\kappa(\gamma)}$ for $\gamma\in\Gamma$. Further $\chi_\gamma=\chi_{\kappa(\gamma)}$ for $\gamma\in\Gamma$, and $\mathrm{Ind}_{T_{1\gamma}}^T \rho_{T_{1\gamma}}^\gamma \cong \mathrm{Ind}_{T_{2\delta}}^T \rho_{T_{2\delta}}^\delta$ for $\gamma\in\Gamma_f$ and $\delta=\kappa(\gamma)$.*