33. *Prime Producing Quadratic Polynomials and Class-number One Problem for Real Quadratic Fields*

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Let $F=Q(\sqrt{m})$ ($m>0$: square-free integer) be a real quadratic field. Denote by $h=h(m)$ and $d=d(m)$ the class number in the wide sense and the discriminant of F, respectively. Recently the following theorem was obtained by Yokoi [4] and Louboutin [1]:

Theorem 1 (Yokoi-Louboutin). *Let p be an odd prime.*

In case $m=4p^2+1$, $h(m)=1$ if and only if $-n^2+n+p^2$ is prime for any integer n such that $1\leq n<p$.

In case $m=p^2+4$, $h(m)=1$ if and only if $-n^2+n+(p^2+3)/4$ is prime for any integer n such that $1\leq n\leq (p-1)/2$.

In case $m=p(p+4)$, $h(m)=1$ if and only if $-n^2+n+(p^2-1)/4$ is prime for any integer n such that $1\leq n\leq (p+1)/2$.

The purpose of this paper is to improve this theorem, especially concerning the sufficient condition for $h(m)=1$, by using "reduced quadratic irrational", and to prove the following:

Theorem 2. *In case $m=4p^2+1$, $h(m)=1$ if and only if $-n^2+n+p^2$ is prime for any integer n such that $\sqrt{p+1}\leq n\leq p-1$.*

In case $m=p^2+4$, $h(m)=1$ if and only if $-n^2+n+(p^2+3)/4$ is prime for any integer n such that $\sqrt{(p+5)/2}\leq n\leq (p-1)/2$.

In case $m=p(p+4)$, $h(m)=1$ if and only if $-n^2+n+p+(p^2-1)/4$ is prime for any integer n such that $\sqrt{(p+1)/2}\leq n\leq (p-1)/2$.

To prove Theorem 2, we need some preliminaries.

For two quadratic irrational numbers α, β, we say that they are *equivalent* to each other and denote $\alpha\sim\beta$ if and only if the periodic part in the expansion of α into a continued fraction is equal to that of β. Moreover, we say that α is *reduced* if and only if $\alpha>1>-\alpha'>0$, where α' is conjugate of α over Q. Then it is well-known that α is reduced if and only if the expansion of α into a continued fraction is purely periodic (cf. Perron [2]).

Put $R(m)=\{\alpha\in Q(\sqrt{m}): \alpha=(b+\sqrt{d})/2a\ (a, b\in N), \alpha \text{ is reduced}\}$. Then it is easily verified that $(d_0+\sqrt{d})/2$ belong to $R(m)$, if we choose $d_0\in N$ satisfying $d_0<\sqrt{d}<d_0+2$ and $d_0\equiv d \bmod 2$.

Now we can obtain the following three lemmas:

Lemma 1. *Set $(d_0+\sqrt{d})/2=[\overline{a_1, a_2, \cdots, a_n}]$, then $h(m)=1$ if and only if $R(m)=\{[\overline{a_i, a_{i+1}, \cdots, a_n, a_1, \cdots, a_{i-1}}]: 1\leq i\leq n\}$.*

Proof. This lemma follows easily from $h(m)=\#(R(m)/\sim)$ (cf. Yamamoto [3]).