1. Functorial Properties of Second Analytic Wave Front Sets and Equivalence of Two Notions of Second Microlocal Singularities

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1. Introduction. This paper aims at studying several functorial properties of second analytic wave front sets of hyperfunctions or microfunctions. As an application we show that two notions of second microlocal singularities, second singular spectrum introduced by M. Kashiwara and second analytic wave front sets due to J. Sjöstrand, are equivalent. To this aim we utilize the partial Radon transformation. We follow the notation prepared in Okada-Tose [11].

2. Second singular spectrum. Let M be, as in § 2 of [11], an open subset in $\boldsymbol{R}^n_x$, and X be a complex neighborhood of M in $\boldsymbol{C}^n_z$. We take coordinates of $T^*_M X(\simeq\sqrt{-1}\,T^*M)$ [resp. T^*M] as $(x\,;\sqrt{-1}\,\xi\cdot dx)$ [resp. $(x\,;\xi\cdot dx)$] with $\xi=(\xi_1,\cdots,\xi_n)$. We identify $\sqrt{-1}\,T^*M$ with T^*M by the correspondence

(2.1) $$(x\,;\sqrt{-1}\,\xi\cdot dx)\longleftrightarrow(x\,;\xi\cdot dx).$$

$T^*_M X$ is endowed with the sheaf $\mathcal{C}_M$ of microfunctions, which enjoys an exact sequence

$$0\longrightarrow\mathcal{A}_M\longrightarrow\mathcal{B}_M\longrightarrow\dot{\pi}_{M*}(\mathcal{C}_M|_{T^*_M X\setminus M})\longrightarrow 0.$$

Here $\mathcal{A}_M$ denotes the sheaf of real analytic functions on M, $\mathcal{B}_M$ that of hyperfunctions, and $\dot{\pi}_M$ the restriction to $\dot{T}^*_M X$ $(\simeq T^*_M X\setminus M)$ of the natural projection $\pi_M: T^*_M X\to M$. Moreover there exists a canonical sheaf morphism

$$Sp_M:\pi_M^{-1}\mathcal{B}_M\longrightarrow\mathcal{C}_M\qquad(\pi_M: T^*_M X\longrightarrow M),$$

by which we set for $u\in\mathcal{B}_M$

$$\mathrm{SS}\,(u)\;:=\mathrm{supp}\,(Sp_M(u)).$$

Then $\mathrm{SS}\,(u)$ is called the singular spectrum of u (refer to [13]). We remark that for $u\in\mathcal{B}_M$, we have

$$\mathrm{SS}\,(u)=\mathrm{WF}_a\,(u)$$

through the correspondence (2.1). This is a classical fact dating back to J.M. Bony [2], K. Kataoka [7]. Refer also to J. Sjöstrand [12].

Now let V denote an involutive submanifold in $T^*_M X$:

$$V=\{(x\,;\sqrt{-1}\,\xi\cdot dx)\,;\,\xi_1=\cdots=\xi_d=0\}.$$

We set

$$N=\{z\in X\,;\,\mathrm{Im}\,z''=0\},\qquad\tilde{V}=T^*_N X\setminus N.$$

We take coordinates of $\tilde{V}$ as $(z',x''\,;\sqrt{-1}\,\xi''\cdot dx'')$, and we have in T^*X the injection

$$V=\dot{T}^*_M X\cap\dot{T}^*_N X\hookrightarrow\tilde{V}.$$