27. *Spectral Properties of the Operator Associated with a Retarded Functional Differential Equation in Hilbert Space*

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In [4] the fundamental result on the structural operator for the linear retarded functional differential equation

$$du(t)/dt = A_0u(t) + A_1u(t-h) + \int_{-h}^{0} a(s)A_2u(t+s)\,ds \tag{1}$$

in a Hilbert space H was established. Here, $-A_0$ is the operator associated with a bounded sesquilinear form $a(u, v)$ defined in $V \times V$ and satisfying Gårding's inequality

$$\operatorname{Re} a(u, u) \geq c\|u\|^2, \quad c > 0,$$

where V is a Hilbert space densely and continuously imbedded in H and $\| \ \|$ is the norm of V. It is known that A_0 generates an analytic semigroup in both of H and V^*. It is assumed that A_1 and A_2 are bounded linear operators from V to V^* and $A_iA_0^{-1}$, $i=1, 2$, are bounded also in H. The real valued function $a(s)$ is assumed to be Hölder continuous in $[-h, 0]$.

Let $S(t): M = H \times L^2(-h, 0; V) \to M$ be the solution semigroup for (1) considered as an equation in V^*: for $g = (g^0, g^1) \in M$

$$S(t)g = (u(t; g), \quad u(t+\cdot\,; g)),$$

where $u(t; g)$ is the mild solution of (1) satisfying the initial conditions

$$u(0; g) = g^0, \quad u(s; g) = g^1(s) \qquad \text{for } s \in [-h, 0). \tag{2}$$

In this paper we investigate the spectral properties of the infinitesimal generator A of $S(t)$ in the special case where $A_1 = \gamma A_0$ with some real constant γ, $A_2 = A_0$ and the imbedding $V \subset H$ is compact. Hence, in what follows throughout this paper we consider the equation

$$du(t)/dt = A_0u(t) + \gamma A_0u(t-h) + \int_{-h}^{0} a(s)A_0u(t+s)\,ds \tag{3}$$

with A_0, γ, a satisfying the assumptions stated above.

According to the Riesz-Schauder theory A_0 has a discrete spectrum: $\sigma(A_0) = \{\mu_j : j = 1, 2, \cdots\}$. Set

$$m(\lambda) = 1 + \gamma e^{-\lambda h} + \int_{-h}^{0} e^{\lambda s} a(s)\,ds. \tag{4}$$

It is clear that $m(\lambda)$ is an entire function and

$$m(\lambda) \to 1 \qquad \text{as } \operatorname{Re} \lambda \to +\infty. \tag{5}$$

The following lemmas are proved as Theorems 6.1 and 7.2 of

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