25. *Period Four and Real Quadratic Fields of Class Number One*

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The purpose of this note is to provide criteria, in terms of prime-producing quadratic polynomials, for a real quadratic field $\boldsymbol{Q}(\sqrt{d})$ to have class number $h(d)=1$, when the continued fraction expansion of ω is 4 (where $\omega=(1+\sqrt{d})/2$ if $d\equiv 1 \pmod 4$ and $\omega=\sqrt{d}$ if $d\equiv 2, 3 \pmod 4$). This continues the work of the first author in [4]–[11] and that of both authors in [12]–[18] in the quest for a general "Rabinowitsch-like" result for real quadratic field. Rabinowitch [19]–[20], proved that if $p\equiv 3 \pmod 4$ is prime then $h(-p)=1$ if and only if $x^2-x+(p+1)/4$ is prime for all integers x with $1\leq x\leq (p-7)/4$, $p>7$. In [4] the first author found such a criterion for real quadratic fields of narrow *Richaud-Degert* (R-D)-type (see [1] and [21]). $\boldsymbol{Q}(\sqrt{d})$ (or simply d) is said to be R-D type if $d=l^2+r$ with $4l\equiv 0 \pmod r$ and $-l<r\leq l$. If $|r|\in\{1, 4\}$ then d is said to be of *narrow* R-D type. In [15]–[16] we found similar criteria for general R-D types. In fact in [18] we completed the task of actually determining *all* real quadratic fields of R-D type having class number one (with possibly only one more value remaining). However, our forging of intimate links between the class number one problem and prime-producing quadratic polynomials makes the existence of the potential additional value virtually impossible.

With the virtual solution of the class number one problem for real quadratic fields of R-D type the authors turned their attention to the general case. In [12] we found a Rabinowitsch criterion for $d\equiv 1 \pmod 4$ where ω has period 3. Several examples of *non*-R-D types were provided as applications. The result in this paper is to find such a criterion when ω has period 4. Moreover for $d\not\equiv 5 \pmod 8$ we determine all such d with class number one (with possibly only one more value remaining).

Theorem 1. *Let square-free* $d\equiv 1$ *(mod 4) and* $\omega=\langle a, \overline{b, c, b, 2a-1}\rangle$ *(the continued fraction expansion of period 4),* $d=(2a-1)^2+4(c(fb-c)+f)$, *and* $2a-1=b^2cf-bc^2+c-2bf$ *for some positive integers* a, b, c *and* f. *Let, furthermore,* $f_d(x)=-x^2-x+(d-1)/4$. *Then* $h(d)=1$ *if and only if the following conditions* (1)–(6) *all hold.*

(1) $b(fb-c)+1$ *is prime.*

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