19. *A Theory of Infinite Dimensional Cycles for Dirac Operators*

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(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 13, 1989)

1. Introduction. We begin with a general theory and apply it to Dirac operators in the last section. Let $\mathcal{F}=\{F_x\}_{x\in X}$ be a family of Fredholm operators parametrized by an infinite dimensional space X. We are interested in a family (not necessarily a bundle) of solutions of this family of operators.

The family of solutions of operators gives rise to an infinite dimensional cycle κ (called *kernel cycle*) which represents a global structure of the family of solutions. We shall estimate this cycle from below by another cycle ψ (called *index cycle*) determined by the index of the family of operators. Using essentially the vanishing theorem of Lichnerowicz [5], we can show this index cycle is non-trivial for Dirac operators. There is a relation between these cycles and a symplectic geometry, which will be mentioned in forthcoming publications.

Our cycles κ and ψ are motivated by the Catastrophe theory developped by R. Thom [8] and E.C. Zeeman [9]. Especially index cycles ψ are closely related to Thom-Boardman singularities (cf. J.M. Boardman [2], F. Ronga [7] and H. Morimoto [6]).

The method to prove the non-triviality of index cycles for Dirac operators is based on the idea of Atiyah-Jones [1]. They proved non-triviality of characteristic cycles χ. We apply their method to index cycles ψ taking into consideration our estimate $\kappa\supset\psi$.

The detailed proofs will be given elsewhere.

This research is partially supported by the British Council. The author is very much grateful to Prof. J. Eells and Prof. K.D. Elworthy for their kind hospitality at Warwick University.

2. General estimates for cycles. Let X be an infinite dimensional paracompact space, and let $\mathcal{F}=\{F_x\}_{x\in X}$ be a continuous family of Fredholm operators $F_x: E\to E'$, $x\in X$, here E and E' are infinite dimensional Hilbert spaces (or more generally Kuiper spaces). First we set,

$$\chi^*{}_{q,p}(\mathcal{F})=\chi_{p,q}(\mathcal{F}^*)=\{x\in X\,;\,\dim(\ker(F_x^*))\geq p\},$$

where p and q are integers with $p-q=k$ and k is the numerical index of $\mathcal{F}$. This cycle was studied in a general situation by U. Koshorke [4] and its non-triviality was shown by Atiyah-Jones [1] for Dirac operators.

We are concerned with important subcycles of $\chi^*_{q,p}$. Take filtrations of the bundle $E\times X$, $\{E_n\}$, $\{E^{\infty-n}\}_{n=1,2,\dots}$ such that $E\times X=E_n\oplus E^{\infty-n}$ for any n.