18. Correspondences for Hecke Rings and l-Adic Cohomology Groups on Smooth Compactifications of Siegel Modular Varieties

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Introduction. We report that the Hecke rings act on the l-adic cohomology groups of suitable non-singular projective toroidal compactifications of the higher dimensional modular varieties. We extend the fixed point theory of Lefschetz to the correspondences for the Hecke rings on those compactifications. We treat here the Siegel modular case. For details see Hatada [6], which will appear elsewhere.

§1. Let $g\geq 1$, $w\geq 0$, $j\geq 1$, $k\geq 1$, and $N\geq 3$ be rational integers. Let $\mathscr{R}$ denote a ring. Write

$M_{j,k}(\mathscr{R})=$the set of $j\times k$ matrices with coefficients in $\mathscr{R}$; $\mathscr{R}^j=M_{1,j}(\mathscr{R})$;

$1_k=$the $k\times k$ unit matrix $\in M_{k,k}(\boldsymbol{Z})$; $J_g=\begin{bmatrix}0 & -1_g\\ 1_g & 0\end{bmatrix}\in M_{2g,2g}(\boldsymbol{Z})$;

$\mathfrak{H}_g=$the Siegel upper half plane of degree g

$=\{Z\in M_{g,g}(\boldsymbol{C})\mid Z={}^tZ.\ \mathrm{Im}\,Z$ is positive definite.$\}$;

$\mathrm{Sp}(g,\boldsymbol{Z})=$the full symplectic modular group $\subset M_{2g,2g}(\boldsymbol{Z})$;

$\mathrm{GSp}^+(g,\boldsymbol{R})=\{\gamma\in\mathrm{GL}(2g,\boldsymbol{R})\mid {}^t\gamma J_g\gamma J_g^{-1}$ is a scalar matrix whose eigenvalue is positive.$\}$; $\mathrm{GSp}^+(g,\boldsymbol{Z})=\mathrm{GSp}^+(g,\boldsymbol{R})\cap M_{2g,2g}(\boldsymbol{Z})$;

$r(\alpha)=$the eigenvalue of ${}^t\alpha J_g\alpha J_g^{-1}$ for $\alpha\in\mathrm{GSp}^+(g,\boldsymbol{R})$;

$\mathrm{GSp}^+(g,\boldsymbol{R})\ltimes\boldsymbol{R}^{2gw}=$the semi-direct product of $\mathrm{GSp}^+(g,\boldsymbol{R})$ and $\boldsymbol{R}^{2gw}$ with $\boldsymbol{R}^{2gw}$ normal such that

$$(\alpha,\boldsymbol{m})\cdot(\beta,\boldsymbol{n})=\left(\alpha\cdot\beta,\ r(\beta)^{-1}\boldsymbol{m}\begin{bmatrix}\beta & & & 0\\ & \beta & & \\ & & \ddots & \\ 0 & & & \beta\end{bmatrix}+\boldsymbol{n}\right)$$

for all $\boldsymbol{m}$ and $\boldsymbol{n}\in\boldsymbol{R}^{2gw}$ and all α and $\beta\in\mathrm{GSp}^+(g,\boldsymbol{R})$. (In the right side the products are those for matrices.) We let $\mathrm{GSp}^+(g,\boldsymbol{R})\ltimes\boldsymbol{R}^{2gw}$ act on the complex analytic space $\mathfrak{H}_g\times\boldsymbol{C}^{gw}=\{(Z,\xi_1,\xi_2,\cdots,\xi_w)\mid Z\in\mathfrak{H}_g,\ \xi_j\in\boldsymbol{C}^g$ for any $j\in[1,w].\}$ to the left as follows. Write $\boldsymbol{m}=(\boldsymbol{m}_1,\boldsymbol{n}_1,\boldsymbol{m}_2,\boldsymbol{n}_2,\cdots,\boldsymbol{m}_w,\boldsymbol{n}_w)\in\boldsymbol{R}^{2gw}$ with $\boldsymbol{m}_j\in\boldsymbol{R}^g$ and $\boldsymbol{n}_j\in\boldsymbol{R}^g$ for any $j\in[1,w]$, and write $\alpha=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\in\mathrm{GSp}^+(g,\boldsymbol{R})$ partitioned into blocks on dimension $g\times g$. Then

$$\left(\begin{pmatrix}A & B\\ C & D\end{pmatrix},(\boldsymbol{m}_1,\boldsymbol{n}_1,\boldsymbol{m}_2,\boldsymbol{n}_2,\cdots,\boldsymbol{m}_w,\boldsymbol{n}_w)\right)(Z,\xi_1,\xi_2,\cdots,\xi_w)$$
$$=\left((AZ+B)(CZ+D)^{-1},\quad r\left(\begin{pmatrix}A & B\\ C & D\end{pmatrix}\right)\left(\xi_1+(\boldsymbol{m}_1,\boldsymbol{n}_1)\begin{pmatrix}Z\\ 1_g\end{pmatrix}\right)(CZ+D)^{-1},\right.$$