16. Quantum Orthogonal and Symplectic Groups and their Embedding into Quantum GL

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We use quantum R matrices [3] to define quantum orthogonal and symplectic groups in the same way as quantum GL and SL of type A [2, 4, 7]. We also consider embedding the quantum orthogonal and symplectic groups $O_q(n)$ and $Sp_q(n)$ into some q-analogues of $GL(n)$. It seems difficult to embed into $GL_q(n)$ of type A. We suggest there are two other types (orthogonal and symplectic) of q-analogues of $GL(n)$, and explain the embedding of $O_q(3)$ into $GL_q^o(3)$, the quantum $GL(3)$ of orthogonal type, in detail.

We work over a field k, and fix an element $q\neq 0$ in k. Let $\mathcal{M}_n$ be the free associative k-algebra on indeterminates x_{ij}, $i, j=1, \cdots, n$, with the following bialgebra structure:

$$\Delta(x_{ik})=\textstyle\sum_j x_{ij}\otimes x_{jk}, \qquad \varepsilon(x_{ik})=\delta_{ik}.$$

Let X denote the $n\times n$ matrix (x_{ij}) with entries in $\mathcal{M}_n$.

1. Quantum orthogonal groups. For $1\leq i\leq n$, put $i'=n+1-i$ and

$$\bar{i}=\begin{cases} i-(n/2) & \text{if } i<i', \\ 0 & \text{if } i=i', \\ i-(n/2)-1 & \text{if } i>i'. \end{cases}$$

We assume q has a square root $q^{1/2}$ in k when n is odd. Let T denote the following symmetric $n^2\times n^2$ matrix.

$$q\sum_{i\neq i'} e_{ii}\otimes e_{ii}+\sum_{i\neq j,j'} e_{ij}\otimes e_{ji}+(q-q^{-1})\sum_{i<j,i\neq j'} e_{jj}\otimes e_{ii}+\sum_{i'\leq k} a_{ik}e_{ik}\otimes e_{i'k'}$$

where e_{ij} denote matrix units and

$$a_{ik}=\begin{cases} 1 & \text{if } i=i'=k, \\ q^{-1} & \text{if } i\neq i'=k, \\ (q-q^{-1})(\delta_{ik}-q^{-\bar{i}-\bar{k}}) & \text{if } i'<k. \end{cases}$$

We have

$$(T-q)(T+q^{-1})(T-q^{1-n})=0.$$

Definition 1. Define bialgebras $M_q(n)$ and $A_q(n)$ by

$$M_q(n)=\mathcal{M}_n/(X^{(2)}T=TX^{(2)}), \qquad A_q(n)=M_q(n)/(XX'=I=X'X),$$

where $X^{(2)}=(X\otimes I)(I\otimes X)$, and $X'=(q^{\bar{j}-\bar{i}}x_{j'i'})_{ij}$.

Proposition 2. (a) *$A_q(n)$ is a Hopf algebra,* i.e., *has an antipode.*

(b) *If $q\neq\pm 1$, there is a central group-like element γ in $M_q(n)$ such that $XX'=\gamma I=X'X$. The localization $M_q(n)[\gamma^{-1}]$ (with γ^{-1} group-like) is a Hopf algebra, and $A_q(n)$ coincides with the quotient Hopf algebra*

$$M_q(n)/(\gamma-1).$$