14. *Global Behavior of Solutions of Quasilinear Ordinary Differential Systems*

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1. Introduction. Various kinds of sufficient conditions for the asymptotic behavior of solutions of the quasilinear ordinary differential system

(N) $$x'=A(t,x)x+F(t,x)$$

are obtained by Kartsatos (see Chapter 8 in [2]), where $A(t,x)$ is a real $n\times n$ matrix continuous on $\boldsymbol{R}^+\times\boldsymbol{R}^n$, $\boldsymbol{R}^+=[0,+\infty)$, and $F(t,x)$ is an $\boldsymbol{R}^n$-valued function continuous on $\boldsymbol{R}^+\times\boldsymbol{R}^n$.

Together with the above system, the following linear system

(L) $$x'=B(t)x$$

is concerned, where $B(t)$ is a real $n\times n$ matrix continuous on $\boldsymbol{R}^+$.

Hypothesis 1. *The zero solution of* (L) *is uniformly asymptotically stable.*

Hypothesis 1 holds if and only if the zero solution of (L) is globally exponential-asymptotically stable (see [5]).

In order to investigate the global behavior of solutions of (N), Schauder's fixed point theorem will be applied under the following hypothesis.

Hypothesis 2. *All the solutions of* (N) *for the initial value problems are uniquely determined.*

Theorem 1, in which sufficient conditions for the globally uniform-asymptotic stability of the zero solution of (N) are given, is a strict extention of the well known result for the case where $A(t,x)\equiv A(t)$ (see Remark). In Theorem 2 the condition on perturbed term which is considered by Lasota and Opial [3] ensures boundedness of all the solutions of (N) and the globally uniform attractivity of the zero solution of (N). Moreover in Theorem 3, sufficient conditions for the globally exponential-asymptotic stability of the zero solution of $x'=A(t,x)x$ are obtained by using Liapunov's second method.

2. Preliminaries. The symbol $\|\cdot\|$ will denote a norm in $\boldsymbol{R}^n$ and the corresponding norm for $n\times n$ matrices. Let $C(\boldsymbol{R}^+)$ be the space of $\boldsymbol{R}^n$-valued functions continuous on $\boldsymbol{R}^+$ with the supremum norm $\|\cdot\|_\infty$.

Lemma 1. *Hypothesis* 1 *holds if and only if there exist* $K\geqq 1$ *and* $\lambda>0$ *such that*

(1) $$\|X_B(t)X_B^{-1}(\tau)\|\leqq K\exp(-\lambda(t-\tau)) \qquad \textit{for } t\geqq\tau,$$