73. *Theta Series and the Poincaré Divisor*

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Let H_n be the Siegel upperhalf space of degree n, that is, $H_n = \{z \in M_n(C) \mid {}^t z = z,\ \mathcal{I}m\, z > 0\}$. Then the classical theta $\vartheta\begin{bmatrix} k' \\ k'' \end{bmatrix}(z \mid x)$ may be regarded as a function of (z, k', k'', x) on $H_n \times R^n \times R^n \times C^n$. Now we introduce a complex variable $k = zk' + k''$, and after a minor modification of $\vartheta\begin{bmatrix} k' \\ k'' \end{bmatrix}(z \mid x)$, we define a new series $\vartheta(z, k, x)$, which represents a holomorphic function on the space $H_n \times C^n \times C^n$ whose second factor C^n will be regarded as the dual space of the third factor C^n in a natural way. This new function $\vartheta(z, k, x)$ substitutes for the classical theta and sometimes has an advantage because of its complex analyticity. For instance, using this function we can explicitly write down a theta function whose divisor is the Poincaré divisor.

1. The dual lattice. Let (E, G) be a pair of n-dimensional C-vector space E and a lattice subgroup G. Assume that the quotient E/G is an abelian variety, or equivalently that there are a C-basis $(\mathfrak{e}_1, \cdots, \mathfrak{e}_n)$ and an R-basis $(\mathfrak{f}_1, \cdots, \mathfrak{f}_{2n})$ of E such that $(\mathfrak{f}_1, \cdots, \mathfrak{f}_{2n}) = (\mathfrak{e}_1, \cdots, \mathfrak{e}_n)(z\ \ 1_n)$ with a matrix z in the Siegel upperhalf space H_n and the identity n-matrix 1_n (which is sometimes denoted simply by 1), and that G is generated by $(\mathfrak{e}_1, \cdots, \mathfrak{e}_n)(z\ \ e)$ with an $(n \times n)$-matrix e having Z-coefficients and $\det e \neq 0$. Under this C-basis, E is identified with C^n and G is generated by the column vectors of $(z\ \ e)$, denoted by $G = \langle z\ \ e \rangle$. The R-coordinates $\boldsymbol{x} = \begin{pmatrix} x' \\ x'' \end{pmatrix}$, x' and $x'' \in R^n$, of a point $x \in C^n$ under the latter basis are determined by $x = (z\ \ 1_n)\boldsymbol{x} = zx' + x''$.

The classical theta series $\vartheta\begin{bmatrix} k' \\ k'' \end{bmatrix}(z \mid x)$ is defined by

$$\vartheta\begin{bmatrix} k' \\ k'' \end{bmatrix}(z \mid x) = \sum_{r \in Z^n} e\left(\frac{1}{2}{}^t(r+k')z(r+k') + {}^t(r+k')(x+k'')\right),$$

where (z, k', k'', x) are variables on $H_n \times R^n \times R^n \times C^n$, and for each $s = (z\ \ 1)\begin{pmatrix} s' \\ s'' \end{pmatrix}$, s', $s'' \in Z^n$, we have

$$\vartheta\begin{bmatrix} k' \\ k'' \end{bmatrix}(z \mid x+s) = \vartheta\begin{bmatrix} k' \\ k'' \end{bmatrix}(z \mid x) e\left(-{}^t s' x - \frac{1}{2}{}^t s' z s' - {}^t k'' s' + {}^t k' s''\right),$$

which suggests that $\begin{pmatrix} -k'' \\ k' \end{pmatrix}$ should be regarded as the R-coordinates of a point $\mathfrak{k}$ of the dual space $\hat{E} = \mathrm{Hom}_R(E, C)/\mathrm{Hom}_C(E, C)$ of $E = C^n$, which is naturally identified with $\mathrm{Hom}_R(E, R)$ by the restriction of the projection