71. A Note on the Universal Power Series for Jacobi Sums

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§ 1. **Introduction.** This note is a supplement of our previous work [5], and we use the same notation as in [5].

Let l be a fixed odd prime number. Ihara [7] constructed for each element ρ of $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q}(\mu_{l^\infty}))$ an l-adic two variable power series $F_\rho(u, v)$ by using a tower of Fermat curves. Some properties of $F_\rho(u, v)$ were studied by [7], Anderson [1], Coleman [3], Ihara-Kaneko-Yukinari [8], etc. In particular, it is proved that the power series $F_\rho(u, v)$ is universal for Jacobi sums and "hence" can be written as a product of three copies of a certain one variable power series. We denote by $g_\rho(t)$ the "twisted log" of the one variable power series, which is known to be an element of $\boldsymbol{Z}_l[[t]]$ (cf. [8]).

The purpose of this note is to describe the difference (if any) between the "expected" image of the homomorphism

$$\tilde{\boldsymbol{g}}:\ \mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q}(\mu_{l^\infty})) \ni \rho \longrightarrow g_\rho(t) \bmod l \in \boldsymbol{F}_l[[t]]$$

and its actual image by means of Iwasawa invariants of the l-cyclotomic field $\boldsymbol{Q}(\mu_{l^\infty})$.

To be more precise, denote by $\tilde{\mathcal{V}}^-$ the additive group consisting of all the power series $g(t)$ in $\boldsymbol{F}_l[[t]]$ satisfying

$$D^{l-1}g = g \quad \text{and} \quad g((1+t)^{-1}-1) = -g(t).$$

Here, $D=(1+t)d/dt$ is a differential operator on $\boldsymbol{F}_l[[t]]$. Then, this module $\tilde{\mathcal{V}}^-$ is the "expected" image in the following sense:

Theorem 1 ([5, Th. 3′]). *$\mathrm{Im}\,\tilde{\boldsymbol{g}} \subset \tilde{\mathcal{V}}^-$, and both sides coincide if and only if the Vandiver conjecture is valid.*

Let λ be Iwasawa's λ-invariant of the cyclotomic $\boldsymbol{Z}_l$-extension of the real cyclotomic field $\boldsymbol{Q}(\cos(2\pi/l))$. In § 2, we define an invariant ε of a certain Galois group over $\boldsymbol{Q}(\mu_{l^\infty})$, which is very similar to its ν-invariant. Our result is

Theorem 2. *The cardinality of the quotient $\tilde{\mathcal{V}}^-/(\mathrm{Im}\,\tilde{\boldsymbol{g}})$ is finite and is equal to $l^{\lambda+\varepsilon}$.*

On the other hand, Coleman [3] proved that the power series $g_\rho(t)$ satisfies some non obvious functional equations and that these functional equations characterize the image of the homomorphism

$$\boldsymbol{g}:\ \mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q}(\mu_{l^\infty})) \ni \rho \longrightarrow g_\rho(t) \in \boldsymbol{Z}_l[[t]]$$

if and only if the Vandiver conjecture is valid. In [5, Th. 2], we described the difference between the "expected" image of $\boldsymbol{g}$ and its actual image by means of Iwasawa type invariant of $\boldsymbol{Q}(\mu_{l^\infty})$. Theorems 1 and 2 are modulo l version of these results.