32. *Some Qualitative Aspects of Transversely Projective Foliations*

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§ 1. **Introduction.** Let G be a Lie group which acts transitively and real analytically on a q-dimensional real analytic manifold X. A codimension q foliation $\mathcal{F}$ on a manifold M is called a transversely (G, X) foliation if M is covered by a collection of $\mathcal{F}$-distinguished charts with submersions to X for which the transition functions are given by the action of G. They form a special class of foliations which are, for example, rigid in some sense ([3], [9]). They are sources of examples to show the complexity and the diversity of foliations mainly in quantitative points of view. In particular, various secondary characteristic classes are shown not to vanish on suitable transversely (G, X) foliations (see, e.g., [2], [6], [8], [12], [18]).

On the other hand, to understand the qualitative nature of transversely (G, X) foliations is another interesting problem. There are a series of investigations ([1], [7], [11], [13], [16], [17]) to this end for *transversely affine foliations*, i.e. when $(G, X)=(\mathrm{Aff}^+(\boldsymbol{R}), \boldsymbol{R})$, where $\mathrm{Aff}^+(\boldsymbol{R})$ is the group of orientation preserving affine transformations on $\boldsymbol{R}$. One has a considerably good grip on the geometric aspects of transversely affine foliations.

The purpose of the present paper is to provide basic knowledge about the geometric nature of transversely projective foliations. A *transversely projective foliation* is a transversely (G, X) foliation for $(G, X)=(\mathrm{PSL}(2, \boldsymbol{R}), S^1_\infty)$, where $\mathrm{PSL}(2, \boldsymbol{R})=\mathrm{SL}(2, \boldsymbol{R})/\{\pm I\}$ acts on $S^1_\infty=\boldsymbol{R}\cup\{\infty\}$ as linear fractional transformations. Transversely projective foliations constitute a far wider class, and are far more intriguing, than transversely affine foliations. In fact, any transversely oriented C^2 foliation without compact leaves on the unit tangent bundle of a closed oriented surface of genus >1 is transversely projective up to topological conjugacy ([14]). Transversely projective foliations are known to support the Godbillon-Vey class ([8]), while transversely affine foliation do not ([7]).

Given a transversely (G, X) foliations $\mathcal{F}$ on a manifold M, one can associate by means of analytic continuation a developing submersion

$$D : \tilde{M} \to X,$$

where $\tilde{M}$ is the universal covering of M, and a global holonomy homo-

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