28. The Behaviour near the Characteristic Surface of Singular Solutions of Linear Partial Differential Equations in the Complex Domain

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Let $L(z,\partial_z)$ be a linear partial differential operator with the order $m\geq 1$. Its coefficients are holomorphic in a neighbourhood of the origin $z=0$ in C^{n+1}. K is a nonsingular complex hypersurface through $z=0$. In the present paper we treat the equation

$$L(z,\partial_z)u(z)=f(z). \tag{0.1}$$

We assume K is characteristic for $L(z,\partial_z)$. The functions $u(z)$ and $f(z)$ in (0.1) are holomorphic except on K. The results are the following: If $u(z)$ has some growth order near K and the behaviour of $f(z)$ near K is mild, then that of $u(z)$ is also the same type. (Theorems 2.1 and 2.3 and Corollaries). The proofs will be given elsewhere.

§ 1. **Definitions.** In order to state the results we give notations and definitions: $z=(z_0,z_1,\cdots,z_n)=(z_0,z')$ is the coordinate of C^{n+1}. $|z|=\max\{|z_i|;\ 0\leq i\leq n\}$. $\partial_z=(\partial_0,\partial_1,\cdots,\partial_n)=(\partial_0,\partial')$, $\partial_i=\partial/\partial z_i$. We choose the coordinate so that $K=\{z_0=0\}$. We can write the operator $L(z,\partial_z)$ in the form

$$\begin{cases} L(z,\partial_z)=\sum_{k=0}^{m}L_k(z,\partial_z), \\ L_k(z,\partial_z)=\sum_{l=s_k}^{k}A_{k,l}(z,\partial')(\partial_0)^{k-l}, \\ A_{k,l}(z,\partial')=(z_0)^j a_{k,l}(z,\partial') \quad j=j(k,l), \end{cases} \tag{1.1}$$

where $L_k(z,\partial_z)$ is the homogeneous part of order k, $A_{k,s_k}(z,\partial')\not\equiv 0$ if $L_k(z,\partial_z)\not\equiv 0$ and $a_{k,l}(0,z',\partial')\not\equiv 0$ if $A_{k,l}(z,\partial')\not\equiv 0$. We put $s_k=+\infty$ if $L_k(z,\partial_z)\equiv 0$, and $j=j(k,l)=+\infty$ if $A_{k,l}(z,\partial')\equiv 0$.

Let us define the characteristic indices introduced in Ōuchi [7] and [8]. Put $d_{k,l}=l+j(k,l)$ and

$$d_k=\min\{d_{k,l};\ s_k\leq l\leq k\}. \tag{1.2}$$

Put $A=\{(k,d_k)\in R^2:\ 0\leq k\leq m,\ d_k\neq +\infty\}$. Let $\hat{A}$ be the convex hull of A. Let Σ be the lower convex part of the boundary of $\hat{A}$, and Δ be the set of vertices of Σ, $\Delta=\{(k_i,d_{k_i});\ i=0,1,\cdots,l'\}$, $m=k_0>k_1>\cdots>k_{l'}\geq 0$. We put

$$\sigma_i=\max\{1,(d_{k_{i-1}}-d_{k_i})/(k_{i-1}-k_i)\}. \tag{1.3}$$

Then there exists a $p\in N$ such that $\sigma_1>\sigma_2>\cdots>\sigma_{p-1}>\sigma_p=1$. We call $\{\sigma_i;\ 1\leq i\leq p\}$ the characteristic indices of $L(z,\partial_z)$ for the surface K.

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*) Dedicated to Professor Tosifusa KIMURA on his 60th birthday.