28. *On the Essential Self-adjointness of Pseudo-differential Operators*

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§1. **Introduction.** In a rigorous treatment of quantum mechanics it is basically important to consider the problem: Is a quantum Hamiltonian self-adjoint? In the present paper we state several theorems on the essential self-adjointness of pseudo-differential operators with Weyl symbols. Applying the theorems we can show the essential self-adjointness of Weyl quantized Hamiltonians.

In [8], M. A. Shubin gives a proof of essential self-adjointness of pseudo-differential operators by using a global hypo-elliptic estimate. However, we can obtain the theorems without use of hypo-ellipticity. In order to get our main result we use an algebra of spatially inhomogeneous pseudo-differential operators, which are studied, for example, in [1], [3] and [4].

We do not give detailed proofs of the theorems here. The detailed proofs will be published elsewhere.

§2. **An algebra of pseudo-differential operators.** We give here some results on pseudo-differential operators. The results have already been obtained fundamentally by Iwasaki [3] and Kumano-go and Taniguchi [4], however, we have to reproduce some of their results in a suitable form to our purpose.

Definition 2.1 (see [3] and [4]). A smooth function $\lambda(x,\xi)$ on $\boldsymbol{R}^d\times\boldsymbol{R}^d$ is called a basic weight function if

(1) $1\le\lambda(x+y,\xi)\le C_0\langle y\rangle^{\tau}\lambda(x,\xi)$,

(2) $|\lambda^{(\alpha)}_{(\beta)}(x,\xi)|\le C_{\alpha\beta}\lambda(x,\xi)^{1-|\alpha|+\delta|\beta|}$ for any α and β,

where τ and δ are non-negative constants with $0\le\delta<1$, $\langle y\rangle=(1+|y|^2)^{1/2}$ and

$$\lambda^{(\alpha)}_{(\beta)}(x,\xi)=\left(-i\frac{\partial}{\partial x}\right)^{\beta}\left(\frac{\partial}{\partial\xi}\right)^{\alpha}\lambda(x,\xi).$$

Definition 2.2 (see [3] and [4]). Let m, δ and ρ be real numbers with $0\le\delta<\rho\le 1$. We say that a smooth function $p(x,\xi)$ belongs to the class $S^m_{\lambda,\rho,\delta}$, if $p(x,\xi)$ satisfies

$$|p^{(\alpha)}_{(\beta)}(x,\xi)|\le C_{\alpha\beta}\lambda(x,y)^{m-\rho|\alpha|+\delta|\beta|}\qquad\text{for any }\alpha\text{ and }\beta.$$

Let $\mathcal{S}$ denote the Schwartz space of rapidly decreasing functions on $\boldsymbol{R}^d$. For $p(x,\xi)\in S^m_{\lambda,\rho,\delta}$ we define operators $p(X,D)$ and $p^w(X,D)$ on $\mathcal{S}$ by

$$p(X,D)u(x)=(2\pi)^{-d}\int e^{ix\cdot\xi}p(x,\xi)\hat{u}(\xi)d\xi,$$

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