20. *Tightness Property for Symmetric Diffusion Processes*

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§1. Introduction. Let $\mathcal{E}^n$ be a sequence of closable symmetric forms on $L^2(\mathrm{R}^d, m_n)$ with symmetric non-negative definite (in i, j) measurable coefficients $a^n_{i,j}$:

$$\mathcal{E}^n(f, g)=\frac{1}{2}\sum_{i,j=1}^{d}\int_{\mathrm{R}^d} a^n_{ij}(x)\frac{\partial f}{\partial x_i}(x)\frac{\partial g}{\partial x_j}(x)dm_n$$
$$\mathcal{D}[\mathcal{E}^n]=C_0^\infty(\mathrm{R}^d)$$

where m_n are everywhere dense positive Radon measures and $C_0^\infty(\mathrm{R}^d)$ is the space of infinitely differentiable functions with compact support. We assume that there exists a positive constant c such that

$$\sup_n \sum_{i,j=1}^{d} a^n_{ij}(x)\xi_i\xi_j \leqq c|\xi|^2$$

for all x and $\xi \in \mathrm{R}^d$. Set $\mathcal{E}^n_1(f, g)=\mathcal{E}^n(f, g)+(f, g)_{m_n}$ and denote the $\mathcal{E}^n_1$-closure of C_0^∞ by $\mathcal{F}^n$. Then we have a sequence of regular Dirichlet spaces $(\mathcal{E}^n, \mathcal{F}^n)$ on $L^2(\mathrm{R}^d, m_n)$ and symmetric diffusion processes $\mathrm{M}^n=(P^n_x, X_t)$ associated with $(\mathcal{E}^n, \mathcal{F}^n)$ (see [3]).

For the probability measure μ_n on R^d, we define the probability measure $P^n_{\mu_n}$ on $C([0, \infty))$ as $P^n_{\mu_n}(\cdot)=\int P^n_x(\cdot)d\mu_n$, where $C([0, \infty))$ is the space of all continuous functions from $[0, \infty)$ into R^d. We are concerned with the problem of finding conditions for a sequence $\{P^n_{\mu_n}\}$ to be tight.

§2. Statement of theorem. We consider the following conditions.

Condition 1. Diffusion processes M^n are conservative.

Condition 2. i) $\sup_n m_n(K)<\infty$ for any compact set K

ii) $\mu_n=\phi_n dm_n$ and $\sup_n \|\phi_n\|_\infty<\infty$

iii) $\{\mu_n\}$ is tight

Condition 3. For any $T>0$ and $R>0$

$$\sup_n \sum_{k=0}^{\infty} m_n(T_{R+k})l^{1/2}\left(\frac{k}{\sqrt{dcT}}\right)<\infty$$

where

$$T_p=\{x \in \mathrm{R}^d ; p\leqq|x|<p+1\} \quad \text{and} \quad l(a)=\frac{1}{\sqrt{2\pi}}\int_a^\infty e^{-u^2/2}du.$$

Then, we have

Theorem. *Under Conditions* 1, 2 *and* 3, *the sequence of probability measure* $\{P^n_{\mu_n}\}$ *is tight.*

Remark 1. Under Condition 1 and Condition 2-i), ii), Lyons-Zheng [4]