19. On the Representation of the Scattering Kernel for the Elastic Wave Equation

By Hideo SOGA
Faculty of Education, Ibaraki University

(Communicated by Kôsaku YOSIDA, M. J. A., March 14, 1988)

Introduction. In Yamamoto [7] and Shibata and Soga [4] we have known that we can construct the scattering theory for the elastic wave equation corresponding to the theory for the scalar-valued wave equation formulated by Lax and Phillips [1, 2]. On Lax and Phillips' formulation Majda [3] obtained a representation of the scattering kernel (operator), which is very useful for consideration on the inverse scattering problems (cf. Majda [3], Soga [5, 6], etc.). In the present note we shall give the similar representation of the scattering kernel for the elastic wave equation considered in Shibata and Soga [4].

§ 1. **Main results.** Let Ω be an exterior domain in $\boldsymbol{R}_x^n$ $(x=(x_1, \cdots, x_n))$ whose boundary $\partial\Omega$ is a compact C^∞ hypersurface. Throughout this note we assume that the dimension n is odd and $\geqq 3$. Let us consider the elastic wave equation

$$(1.1)\qquad \begin{cases} \left(\partial_t^2 - \sum_{i,j=1}^n a_{ij}\partial_{x_i}\partial_{x_j}\right)u(t,x)=0 & \text{in } \boldsymbol{R}\times\Omega, \\ Bu(t,x)=0 & \text{on } \boldsymbol{R}\times\partial\Omega, \\ u(0,x)=f_1(x), \quad \partial_t u(0,x)=f_2(x) & \text{on } \Omega. \end{cases}$$

Here, a_{ij} are constant $n\times n$ matrices whose (p, q)-component a_{ipjq} satisfies

(A.1) $\quad a_{ipjq}=a_{ptjq}=a_{jqip}, \qquad i, j, p, q=1, 2, \cdots, n,$

(A.2) $\quad \sum_{i,p,j,q=1}^n a_{ipjq}\varepsilon_{jq}\bar{\varepsilon}_{ip} \geqq \delta \sum_{i,p=1}^n |\varepsilon_{ip}|^2 \qquad$ for Hermitian matrices (ε_{ij}),

(A.3) $\quad \sum_{i,j=1}^n a_{ij}\xi_i\xi_j$ has characteristic roots of constant multiplicity for $\xi=(\xi_1, \cdots, \xi_n)\in \boldsymbol{R}^n-\{0\}$,

and the boundary operator B is of the form

$$Bu=u|_{\partial\Omega} \quad \text{or} \quad \sum_{i,j=1}^n \nu_i(x)a_{ij}\partial_{x_j}u|_{\partial\Omega},$$

where $\nu=(\nu_1, \cdots, \nu_n)$ is the unite outer vector normal to $\partial\Omega$. We denote by $U(t)$ the mapping: $f=(f_1, f_2)\to(u(t, \cdot), \partial_t u(t, \cdot))$ associated with (1.1), and by $U_0(t)$ the one associated with the equation in the free space $(\Omega=\boldsymbol{R}^n)$.

Under the assumptions (A.1)–(A.3) it has been proved in Shibata and Soga [4] that the wave operators $W_\pm=\lim_{t\to\pm\infty} U(-t)U_0(t)$ are well defined and complete (cf. § 3 of [4]). Let $\{\lambda_j(\xi)\}_{j=1,\dots,d}$ $(\lambda_1<\cdots<\lambda_d)$ be the eigenvalues of $\sum_{i,j=1}^n a_{ij}\xi_i\xi_j$, and let $P_j(\xi)$ be the projection into the eigenspace of $\lambda_j(\xi)$. For the data $f=(f_1, f_2)$ $(\in \mathcal{S})$ in the free space, let us set

$$T_0 f(s, \omega)=\sum_{j=1}^d \lambda_j(\omega)^{1/4}P_j(\omega)(-\lambda_j(\omega)^{1/2}\partial_s^{(n+1)/2}\tilde{f}_1+\partial_s^{(n-2)/2}\tilde{f}_2)(\lambda_j(\omega)^{1/2}s, \omega),$$