17. *Single-point Blow-up for Semilinear Parabolic Equations in Some Non-radial Domains*

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§0. **Introduction.** In this note, we consider

$$\text{(E)}\qquad \begin{cases} u_t = \Delta u + f(u), & (t,x) \in (0,T)\times\Omega, \\ u=0, & (t,x)\in(0,T)\times\partial\Omega, \\ u(0,x)=u_0(x), & x\in\bar{\Omega}. \end{cases}$$

Here $\Omega \subset \boldsymbol{R}^N$ $(N \geqq 2)$ is a bounded domain with smooth boundary and the initial value $u_0 = u_0(x) \geqq 0$ is sufficiently smooth, say, $u_0 \in C^1(\bar{\Omega}) \cap C_0(\bar{\Omega})$. The nonlinear term $f(u)$ satisfies

(0.1) $$f \in C^2(0,\infty)\cap C[0,\infty),\quad f(s)>0 \qquad \text{for } s>0.$$

Let $u = u(t,x)$ be the classical solution of (E). Its existence time T is defined by

(0.2) $$T = \sup\{\tau > 0 \mid u(t,x) \text{ is bounded in } [0,\tau]\times\Omega\}.$$

It is well known that for a large class of f and initial value u_0, the solution $u(t,x)$ may blow up, i.e., $T < +\infty$ and

(0.3) $$\overline{\lim_{t\uparrow T}} \|u(t,\cdot)\|_{L^\infty(\Omega)} = +\infty.$$

In this case we say that $u = u(t,x)$ is a *blow-up solution*, and T is the *blow-up time* (see, for instance [3], [4]).

Here, we consider the blow-up points in some non-radial domains and will give some single-point blow-up results under a weaker hypothesis than the radial symmetry or convexity for Ω.

Definition. The *blow-up set*, or the *set of blow-up points* of $u = u(t,x)$ is defined as

$$S = \{x \in \bar{\Omega} \mid \text{there is a sequence } (t_n, x_n) \text{ in } (0,T)\times\Omega \text{ such that } t_n \uparrow T,\ x_n \to x \text{ and } u(t_n,x_n)\to\infty \text{ as } n\to+\infty\},$$

and each point $x \in S$ is called a *blow-up point* of $u(t,x)$.

By the definition, we can see that S is a closed set. The standing assumption throughout this note is that $f(\cdot)$ and u_0 is such that the solution blows up. For f we assume the following condition.

(F) There exists a function $F = F(u)$ such that

(i) $F(s) > 0$, $F'(s) \geqq 0$ and $F''(s) \geqq 0$ for $s > 0$;

(ii) $\displaystyle\int_1^\infty \frac{ds}{F(s)} < +\infty$;

(iii) there is a constant $\sigma > 0$ such that
$$f'(s)F(s) - f(s)F'(s) \geqq \sigma F(s)F'(s) \qquad \text{for } s>0.$$

This condition is originally introduced in [6]. It can be seen that

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