16. *Class Number One Problem for Real Quadratic Fields*

(The conjecture of Gauss)

By Hideo YOKOI
Department of Mathematics, College of General Education,
Nagoya University

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The following conjecture of Gauss on the class number of real quadratic fields is well known:

(G_1): There exist infinitely many real quadratic fields of class number one, or more precisely

(G_2): There exist infinitely many real quadratic fields $Q(\sqrt{p})$ of class number one such that p is prime congruent to 1 mod 4.

In relation to this conjecture of Gauss, the following conjecture of S. Chowla and analogous conjecture of Yokoi are known[1]:

(C_1) (S. Chowla): Let D be a square-free rational integer of the form $D=4m^2+1$ for natural number m. Then, there exist exactly 6 real quadratic fields $Q(\sqrt{D})$ of class number one,

i.e. $(D, m)=(5, 1), (17, 2), (37, 3), (101, 5), (197, 7), (677, 13)$.

(C_2) (H. Yokoi): Let D be a square-free rational integer of the form $D=m^2+4$ for natural number m. Then, there exist exactly 6 real quadratic fields $Q(\sqrt{D})$ of class number one,

i.e. $(D, m)=(5, 1), (13, 3), (29, 5), (53, 7), (173, 13), (293, 17)$.

Concerning the conjectures (C_1), (C_2), R. A. Mollin says[2]: Conjecture (C_1) was proved under the assumption of the generalized Riemann hypothesis in [6], and conjecture (C_2) also can be proved under the same assumption in a similar way.

On the other hand, H. K. Kim, M. G. Leu and T. Ono[3] recently proved that at least one of the two conjectures (C_1), (C_2) is true and that for the other case there are at most 7 quadratic fields $Q(\sqrt{D})$ of class number one by using results of Tatuzawa [1], Yokoi [3] and by the help of a computer.

Let $\varepsilon_D=(1/2)(t_D+u_D\sqrt{D})>1$ be the fundamental unit of the real quadratic field $Q(\sqrt{D})$ for a positive square-free integer D. Then, (C_1) is a conjecture on real quadratic fields $Q(\sqrt{D})$ with $u_D=2$, and (C_2) is a conjecture on real quadratic fields $Q(\sqrt{D})$ with $u_D=1$.

In this paper, we shall prove first the following theorem on real

1) cf. S. Chowla and J. Friedlander [2] and H. Yokoi [3].
2) cf. R. A. Mollin [4].
3) cf. H. K. Kim, M. G. Leu and T. Ono [5].