95. A Note on Isocompact wM Spaces and Mappings

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Introduction. T_2 isocompact wM spaces behave well like T_2 paracompact M spaces. For example, if $f: X \to Y$ is a closed, continuous map of a T_2 isocompact wM space X onto Y, then $Y = \cup_{n \geqslant 0} Y_n$, where, for each $n \geqslant 1$, Y_n is discrete in Y and $f^{-1}(y)$ is compact for each $y \in Y_0$. As such, we investigate some interesting properties of such spaces and their images under nice maps. Refer [5], [1], [4], [2] and [3] respectively, for the notions of q, point countable and countable type, wM, isocompactness, and quasi-G_δ diagonal.

Main section. Theorem 1. (i) *A T_1 space X of point countable type is a q space.* (ii) *A regular isocompact q space X is point countable type.*

Proof of (i). Let $x \in X$ and K be a compact subset of X of countable character with $x \in K$. Let $\{U_n | n \geqslant 1\}$ be a decreasing local base at K. To claim that $\{U_n\}_n$ is a q sequence at x, let $x_n \in U_n$ for each n. Suppose $\{x_n\}_n$ does not cluster. Then, $D = \{x_n | n \geqslant 1\}$ is closed. Assume $K \cap D = \emptyset$. Then, $X - D$ is an open nhd of K. Since, $U_n \not\subset X - D$ for each n, we have a contradiction.

Proof of (ii). Let $x \in X$ and $\{U_n\}_n$ be a q sequence at x with $\overline{U}_{n+1} \subset U_n$ for each n. Let $C(x) = \cap_n U_n$. It follows that $C(x)$ is of countable character and $x \in C(x)$. Therefore X is of point countable type. Q.E.D.

Theorem 2. *If a regular space X with quasi-G_δ diagonal is a q space or a space of point countable type, then the space is first countable.*

Proof. By the Theorem 1 (i), X is a q space in either case. Let $\{\mathcal{U}_n\}_n$ be a quasi-G_δ diagonal sequence. Let $x \in X$, $\{G_n\}_n$ be a q sequence at x and $\{n_k\}_k$ be the strictly increasing sequence of natural numbers with $x \in St(x, \mathcal{U}_n) = \cup \{U \in \mathcal{U}_n | x \in U\}$, iff $n = n_k$ for some $k \leqslant n$. By induction, we can obtain a sequence $\{H_m\}_m$ of open sets with $x \in H_{m+1} \subset \overline{H}_{m+1} \subset H_m \cap G_{m+1} \cap U_{n_{m+1}}$ for each m, where $x \in U_{n_m} \in \mathcal{U}_{n_m}$. It follows that $\{H_m | m \geqslant 1\}$ is a local base at x. Q.E.D.

Corollary 2.1. *If a T_2 wM space with quasi-G_δ diagonal is a quotient image of a locally compact, separable and metrizable space, then the space is locally compact, separable and metrizable.*

Proof. Apply the Theorem 2 and a result of A. H. Stone [7]. Q.E.D.

Theorem 3. *A T_2 isocompact wM space X is countable type.*

Proof. Let $\{\mathcal{U}_n\}_n$ be a decreasing wM sequence and $K \subset X$ be compact. Let $\mathcal{W}_1$ be a finite subcollection of $\mathcal{U}_1$ with $K \subset W_1 = \cup \mathcal{W}_1$. Let $\mathcal{W}'_2$ be an open collection with $K \subset \cup \mathcal{W}'_2$ such that $\overline{\mathcal{W}}'_2 = \{\overline{W} | W \in \mathcal{W}'_2\}$ refines $\mathcal{W}_1 \wedge \mathcal{U}_2$