73. *Indistinguishability of Conjugacy Classes of the Pro-l Mapping Class Group*

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Introduction. Let l be a fixed prime number and $\pi^{(g)}$ denote the pro-l completion of the topological fundamental group of a compact Riemann surface of genus $g\geq 2$. So, we have

$$\pi^{(g)}=F/N,$$

where F is the free pro-l group of rank $2g$ generated by $x_1, \cdots, x_{2g}$ and N is the closed normal subgroup of F which is normally generated by $[x_1, x_{g+1}]\cdots[x_g, x_{2g}]$, $[\ ,\ ]$ being the commutator; $[x, y]=xyx^{-1}y^{-1}$ $(x, y\in F)$. We denote by Γ_g the outer automorphism group of $\pi^{(g)}$ and call it the pro-l mapping class group. Let

$$\lambda : \Gamma_g \longrightarrow \mathrm{GSp}(2g, \boldsymbol{Z}_l)$$

be the canonical homomorphism induced by the action of Γ_g on $\pi^{(g)}/[\pi^{(g)}, \pi^{(g)}]$ (cf. Asada-Kaneko [2, §2]). We treat the case $g=2$. Then, our result is the following

Theorem. *Assume that $l\geq 5$. Then, there exists an integer $N\geq 1$ such that the following statement holds*:

If $A\in \mathrm{GSp}(4, \boldsymbol{Z}_l)$ satisfies the condition $A\equiv 1_4 \bmod l^N$, $\lambda^{-1}(C_A)$ contains more than one Γ_2-conjugacy class. Here, C_A denotes the $\mathrm{GSp}(4, \boldsymbol{Z}_l)$*-conjugacy class containing A.*

In our previous paper [2, § 6], we have proved this "indistinguishability of conjugacy class" under the assumption that $g\geq 3$. The method adopted there is the "calculations modulo $\pi^{(g)}(3)$", which does not seem to work in case $g=2$. ($\{\pi^{(g)}(k)\}_{k\geq 1}$ denotes, as usual, the descending central series of $\pi^{(g)}$.) So, to prove the above theorem, we use the method "calculations modulo $\pi^{(g)}(4)$". Although this requires rather complicated calculations, it is carried out by using the "Lie algebra" of the nilpotent pro-l group $\pi^{(g)}/\pi^{(g)}(4)$.

For those results on the indistinguishability of conjugacy class of the pro-l braid group and the motivation of these studies, see Ihara [3], [4], Kaneko [5].

§ 1. Preliminaries for proving theorem. To prove Theorem, we need some preliminaries. As before, let π $(=\pi^{(2)})$ denote the pro-l completion of the topological fundamental group of a compact Riemann surface of genus 2 and $\tilde{\Gamma}$ denote the automorphism group of π. For an automorphism ρ of π, we put

$$s_i(\rho)=x_i^{\rho}x_i^{-1} \qquad (1\leq i\leq 4).$$