72. *On the Schur Indices of Certain Irreducible Characters of Simple Algebraic Groups over Finite Fields*

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Let G be a connected, reductive linear algebraic group defined over a finite field F_q with q elements of characteristic p and F the corresponding Frobenius endomorphism of G. Let G^F denote the group of F-fixed points of G. In [2] R. Gow initiated, in order to determine the Schur indices of irreducible characters of some finite groups of type G^F, to study rationality-properties of the characters of G^F induced by the linear characters of a Sylow p-subgroup of G^F (also cf. A. Helversen-Passoto [4] and Gow [3]). In [5] we have obtained some general results for a general G^F $(p\neq 2)$. The purpose of this paper is to state some more detailed results when G is a simple algebraic group.

Let G be reductive. Let B and T be respectively an F-stable Borel subgroup of G with the unipotent radical U and an F-stable maximal torus of B. Let R be the set of roots of G with respect to T, R^+ the set of positive roots determined by B and D the set of corresponding simple roots. For each $\alpha\in R$, let U_α denote the corresponding root subgroup of G. Let U_+ be the subgroup of U generated by the U_α, $\alpha\in R^+-D$. There is a permutation ρ on D determined by $FU_\alpha=U_{\rho\alpha}$ for $\alpha\in D$. Let I be the set of orbits of ρ on D. For each $i\in I$, put $U_i=\prod_{\alpha\in i}U_\alpha$. Then we have $U/U_+=\prod_{i\in I}U_i$; this decomposition is F-stable and we have $(U/U_+)^F=U^F/U_+^F=\prod_{i\in I}U_i^F$. It is known that U^F is a Sylow p-subgroup of G^F and that if p is a good prime for G then U_+^F is equal to the commutator subgroup of U^F. Let Λ be the set of characters of U^F such that $\lambda|U_+^F=1$ and let Λ_0 be the set of λ in Λ such that $\lambda|U_i^F\neq 1$ for all $i\in I$. Then it is known that, for any $\lambda\in\Lambda_0$, $\Gamma_\lambda=\mathrm{Ind}_{U^F}^{G^F}(\lambda)$ is multiplicity-free ([1], Theorem 8.1.3; also see [5], Lemma 1). For an irreducible character χ of a finite group and a field E of characteristic zero, let $m_E(\chi)$ denote the Schur index of χ with respect to E. We have seen in [5] that if χ is an irreducible character of G^F such that $\langle\chi,\lambda^{G^F}\rangle_{G^F}=1$ for some $\lambda\in\Lambda$ or that, when p is a good prime for $G, p\nmid\chi(1)$, then we have $m_Q(\chi)\leqq 2$, where Q is the field of rational numbers.

Assume now that G is simple. Let $X=\mathrm{Hom}\,(T,G_m)$ be the (additive) module of rational characters of T. Let $P(R)$ and $Q(R)=\langle R\rangle_Z$ be respectively the weight-lattice and the root-lattice of R, where Z is the ring of rational integers. Then we have $P(R)\supset X\supset Q(R)$; and $P(R)/Q(R)$ is a finite group. Put $d=(X:Q(R))$. For an integer n, let $\mathrm{ord}_2\,n$ denote the exponent