68. Support of CR-hyperfunctions

By Shinichi TAJIMA

Department of Mathematics, Faculty of General Education,
Niigata University

(Communicated by Kôsaku YOSIDA, M. J. A., Sept. 12, 1988)

In this note we examine the possible shape of support of **CR**-hyperfunctions.

Let X be a complex m-dimensional complex manifold, and let N be a real k-codimensional real analytic submanifold of X, with $0 \leq k \leq m$. Throughout this note we assume that the submanifold N is a *generic* **CR**-submanifold (see [2]).
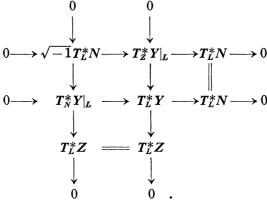
Let $\bar{\partial}_b$ be the tangential Cauchy-Riemann system induced on N. A hyperfunction h on N which satisfies the equations $\bar{\partial}_b h = 0$ is called a **CR**-hyperfunction. We denote by supp (h) the support of **CR**-hyperfunction h.

Remark 1. Every **CR**-hyperfunction defined on a Levi-flat **CR**-submanifold is a hyperfunction with holomorphic parameters (cf. [4]).

Let Y be a complexification of N and let $T_N^* Y (= \sqrt{-1} T^* N)$ be the conormal bundle of N in the cotangent vector bundle $T^* Y$ of Y. Let us denote by $SS(\bar{\partial}_b)$ the characteristic variety of the tangential Cauchy-Riemann system ∂_b.

Note that the purely imaginary locus of the characteristic variety, denoted by $SS(\bar{\partial}_b) \cap T_N^* Y$, is a real $2m$ dimensional manifold (cf. Proposition 1.2.1 of [6]).

Let L be a real analytic submanifold of N and let Z be its complexification. Then we have the following exact sequences (cf. [5]):

$$
\begin{array}{ccc}
0 & & 0 \\
\downarrow & & \downarrow \\
0 \longrightarrow \sqrt{-1}T_L^*N \longrightarrow T_Z^*Y|_L \longrightarrow T_L^*N \longrightarrow 0 \\
\downarrow & & \downarrow & & \| \\
0 \longrightarrow T_N^*Y|_L \longrightarrow T_L^*Y \longrightarrow T_L^*N \longrightarrow 0 \\
\downarrow & & \downarrow \\
T_L^*Z & = & T_L^*Z \\
\downarrow & & \downarrow \\
0 & & 0 \; .
\end{array}
$$

Here we identify $T_N^* Y|_L \cap T_Z^* Y|_L$ with $\sqrt{-1} T_L^* N$.

Definition 2. A real analytic submanifold L of N is said to be totally characteristic, if the purely imaginary conormal bundle $\sqrt{-1} T_L^* N$ satisfies the following condition: