67. *Cyclotomic Invariants for Links*[†),††)]

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In this note we construct numerical link invariants (*cyclotomic invariants*) by using solutions to the star-triangle relation for an N-state IRF model on a two-dimensional square lattice ($N=1,2,\cdots$) [3, 6]. Moreover we will show that these invariants can be defined by using Goeritz matrices and Seifert matrices. We also describe some of their properties; especially relations to the Jones polynomial [5], the Q-polynomial [1, 4], and the Kauffman polynomial [7].

Let $w(a,b,c,d\,;u)$ be the cyclotomic solution described in [6]. We consider a dual graph of an (unoriented) link diagram on a 2-sphere S^2. It decomposes S^2 into some regions and every region can be regarded as a tetragon. So we can assign to each region (or face) the Boltzmann weight $w(a,b,c,d\,;u)$ for every state on the graph as in Fig, 1. Here a state is an assignment of elements in Z/NZ to vertices in the graph.

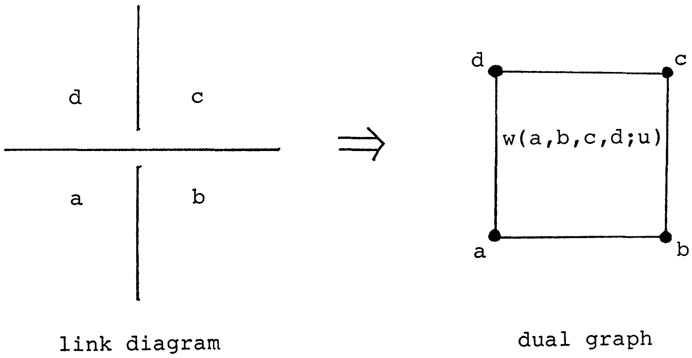

Fig. 1

This is well-defined since $w(a,b,c,d\,;u)=w(c,d,a,b\,;u)$ [6]. If we take the limit $u\to\infty\times\sqrt{-1}$ of $w(a,b,c,d\,;u)$, the partition function $Z_N=\sum\prod w(a,b,c,d\,;u)$ is invariant under the Reidemeister moves $\Omega_3^{\pm1}$ of the link diagram, where the product is taken over all the vertices of the dual graph and the sum is taken over all the states. This follows from the star-triangle relation. See [6, Fig. 2]. See also [2] for the Reidemeister moves.

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