65. *Fractional Powers of Operators and Evolution Equations of Parabolic Type*

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1. Introduction. In 1961 Kato and Sovolevski, one of the creators of the theory, utilized in [2] and [4] respectively the fractional powers of linear operators for studying abstract evolution equations of parabolic type

(E) $du/dt+A(t)u=f(t), \quad 0<t\leq T \quad \text{and} \quad u(0)=u_0$

in a Banach space X. Assuming for some $0<h<1$ independence of t of the definition domains $\mathcal{D}(A(t)^h)$ of the fractional powers $A(t)^h$ and the Hölder condition on $A(t)^h A(s)^{-h}$, they succeeded in formulating an intermediate case between the case of constant domains $\mathcal{D}(A(t))$ and that of completely variable domains. In spite of their beautiful abstract results, generally it is hard to verify the Hölder condition of $A(t)^h A(s)^{-h}$ in applications. Even now the problem remains unsolved except some favorable cases like Hilbert space (cf. [2], [4] and also [6]).

This note is also devoted to formulate the intermediate case but by another condition on $A(t)$, in our condition, the Condition (II) below, the fractional powers do not appear in any explicit form. Apparently the condition seems to be unnatural in form, but it is obtained by linking quite directly the two conditions assumed for handling the two extreme cases, that is, the Hölder condition on $A(t)A(s)^{-1}$ in the case of constant domains and the decay condition of $A(t)(\lambda-A(t))^{-1}dA(t)^{-1}/dt$ in λ in the other case of completely variable. As may be mentioned below, verifying the (II) in applications is now an easy problem. We can remark also that Kato and Sobolevski's condition really implies our condition.

The Condition (II) has been introduced by Acquistapace and Terreni [1] to construct under it the evolution operator $U(t,s)$. But their $U(t,s)$ guarantees existence of the solution of (E) only for regular u_0 and f, and their method of proof seems to be quite complicated. Our method uses the fractional powers of $A(t)$ as before to make the proof simpler and clearer.

$A(t)$, $0\leq t\leq T$, are closed linear operators in a Banach space X (the domains $\mathcal{D}(A(t))$ may not be dense in X). We will make the following assumptions:

(I) The resolvent sets $\rho(A(t))$ of $A(t)$ contain a sector $\sum=\{\lambda\in \boldsymbol{C}; |\arg\lambda|\geq\theta_0\}$ where $0<\theta_0<\pi/2$, and there

$$\|(\lambda-A(t))^{-1}\|_{\mathcal{L}(X)}\leq M/(|\lambda|+1), \quad \lambda\in\sum \qquad \text{with a constant } M\geq 0.$$

(II) For some $0<h, k\leq 1$

$$\|A(t)(\lambda-A(t))^{-1}\{A(t)^{-1}-A(s)^{-1}\}\|_{\mathcal{L}(X)}\leq K|t-s|^k/(|\lambda|+1)^h$$