63. *Solvability in Distributions for a Class of Singular Differential Operators. I*[†]

By Hidetoshi TAHARA
Department of Mathematics, Sophia University

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The local solvability for Fuchsian operators has been studied by many authors (see Tahara [4] and its references). In this paper, the author will establish the local solvability in $\mathcal{D}'$ for a class of (non-Fuchsian) singular hyperbolic operators including

$$L=(t\partial_t)^2-\Delta_x+a(t,x)(t\partial_t)+\langle b(t,x),\partial_x\rangle+c(t,x).$$

§1. Theorem. Let us consider

$$P=(t\partial_t)^m+\sum_{\substack{j+|\alpha|\leq m\\ j<m}} a_{j,\alpha}(t,x)(t\partial_t)^j\partial_x^\alpha,$$

where $(t,x)=(t,x_1,\cdots,x_n)\in \boldsymbol{R}_t\times\boldsymbol{R}_x^n$, $\partial_t=\partial/\partial t$, $\partial_x=(\partial/\partial x_1,\cdots,\partial/\partial x_n)$, $m\in\{1,2,3,\cdots\}$, $\alpha=(\alpha_1,\cdots,\alpha_n)\in\{0,1,2,\cdots\}^n$, $|\alpha|=\alpha_1+\cdots+\alpha_n$ and $\partial_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$. On the coefficients, we assume that $a_{j,\alpha}(t,x)$ ($j+|\alpha|\leq m$ and $j<m$) are C^∞ functions defined in an open neighborhood U of $(0,0)$ in $\boldsymbol{R}_t\times\boldsymbol{R}_x^n$. For $(t,x)\in U$ and $\xi\in\boldsymbol{R}_\xi^n\setminus\{0\}$, denote by $\lambda_1(t,x,\xi),\cdots,\lambda_m(t,x,\xi)$ the roots of the equation (in λ)

$$\lambda^m+\sum_{\substack{j+|\alpha|=m\\ j<m}} a_{j,\alpha}(t,x)\lambda^j\xi^\alpha=0.$$

Assume that for any $(t,x,\xi)\in U\times(\boldsymbol{R}_\xi^n\setminus\{0\})$ the following conditions (H-1)–(H-3) hold:

(H-1) $\lambda_i(t,x,\xi)$ is real for $1\leqq i\leqq m$.

(H-2) $\lambda_i(t,x,\xi)\neq\lambda_j(t,x,\xi)$ for $1\leqq i\neq j\leqq m$.

(H-3) $\lambda_i(t,x,\xi)\neq 0$ for $1\leqq i\leqq m$.

Then we have:

Theorem. *There is an open neighborhood V of $(0,0)$ in $\boldsymbol{R}_t\times\boldsymbol{R}_x^n$ which satisfies the following: for any $f(t,x)$ $(=f)\in\mathcal{D}'(\overline{V})$ there exists a $u(t,x)$ $(=u)\in\mathcal{D}'(\overline{V})$ such that $Pu=f$ holds on V.*

Here, $\overline{V}$ denotes the closure of V and $\mathcal{D}'(\overline{V})$ denotes the set of all distributions defined in a neighborhood of $\overline{V}$.

Remark 1. In the C^∞ function space, the above operator P was already discussed and the following results are known: (1) the local solvability in C^∞ (by Tahara [3], Serra [2]) and (2) the existence of C^∞ null-solutions (by Mandai [1]).

Remark 2. By the same argument given below, we can prove the local solvability in $\mathcal{D}'$ (near the origin) also for the following type of (non-Fuchsian) hyperbolic operators

[†] Dedicated to Professor Tosihusa KIMURA on his 60th birthday.