6. A New Class of Analytic Functions Associated with the Ruscheweyh Derivatives[†]

By H. M. SRIVASTAVA,[*] Shigeyoshi OWA,[**] and O. P. AHUJA[***]

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1. Introduction and definitions. Let $\mathcal{A}(p)$ denote the class of functions of the form

$$f(z)=z^p+\sum_{k=1}^{\infty} a_{p+k}z^{p+k} \qquad (p\in\mathcal{N}=\{1,2,3,\cdots\}) \tag{1.1}$$

which are analytic in the unit disk $\mathcal{U}=\{z:|z|<1\}$. We denote by $f*g(z)$ the Hadamard product (or convolution) of two functions $f(z)\in\mathcal{A}(p)$ and $g(z)\in\mathcal{A}(p)$, that is, if $f(z)$ is given by (1.1) and $g(z)$ is given by

$$g(z)=z^p+\sum_{k=1}^{\infty} b_{p+k}z^{p+k} \qquad (p\in\mathcal{N}), \tag{1.2}$$

then

$$f*g(z)=z^p+\sum_{k=1}^{\infty} a_{p+k}b_{p+k}z^{p+k}. \tag{1.3}$$

Following Goel and Sohi [7], we put

$$D^{n+p-1}f(z)=\frac{z^p}{(1-z)^{n+p}}*f(z) \qquad (n>-p) \tag{1.4}$$

for the $(n+p-1)$th order Ruscheweyh derivative of $f(z)\in\mathcal{A}(p)$.

A function $f(z)\in\mathcal{A}(p)$ is said to be in the class $\mathcal{K}(n,p)$ if and only if

$$\operatorname{Re}\left(\frac{D^{n+p}f(z)}{D^{n+p-1}f(z)}\right)>\frac{n+p}{2(n+1)} \qquad (z\in\mathcal{U}) \tag{1.5}$$

for $n\in\mathcal{N}_0=\mathcal{N}\cup\{0\}$ and $p\in\mathcal{N}$. In particular, for $p=1$, the class $\mathcal{K}(n,1)$ becomes the class $\mathcal{K}_n$ studied by Ruscheweyh [17] who, in fact, proved the basic property [17, p. 110, Theorem 1]:

$$\mathcal{K}_{n+1}\subset\mathcal{K}_n \qquad (n\in\mathcal{N}_0). \tag{1.6}$$

We now introduce the subclass $\mathcal{A}_{n,p}(a,b)$ of $\mathcal{A}(p)$, which is defined below by using the $(n+p-1)$th order Ruscheweyh derivative of $f(z)$.

Definition. Let the function $f(z)$ defined by (1.1) be in the class $\mathcal{A}(p)$, and set

$$F_{n,p}(z)=\frac{D^{n+p}f(z)}{D^{n+p-1}f(z)}-\frac{n+p}{2(n+1)} \tag{1.7}$$

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*) Department of Mathematics, University of Victoria, Victoria, British Columbia V8W 2Y2, Canada.

**) Department of Mathematics, Kinki University, Higashi-Osaka, Osaka, Japan.

***) Department of Mathematics, University of Papua New Guinea, Box 320, University Post Office, Papua New Guinea.

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