38. On the Complex C_Λ Attached to a Certain Class of Lagrangian Set

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0. Introduction. On a real manifold X, we prove that there exist microlocally simple sheaves along some kind of Laglangian set $\Lambda \subset T^*X$, and that such sheaves are unique up to shifts. This has been shown by M. Kashiwara and P. Schapira when Λ is smooth, and used in the course of the study of quantized contact transformations ([2], [3]). In this note we treat some cases where Λ is not smooth as well. As an application we can give a microlocal definition of the complex $C_{\Omega|X}$, which is introduced by P. Schapira [5] for the microlocal study of boundary value problems.

1. Let X be a C^2 manifold, and $\pi: T^*X \to X$ be the cotangent bundle to X. For $F \in Ob(D^+(X))$, the microsupport $SS(F) \subset T^*X$ is defined by Kashiwara-Schapira [1], [3]. For a point $p \in T^*X$, we denote by $D^+(X; p)$ the localization of the category $D^+(X)$ by the null system

$$\mathcal{E}(p) = \{F \in Ob(D^+(X)); SS(F) \not\ni p\}$$

(cf. [2], [3]).

Let Y be a closed submanifold of X, and Ω be an open subset of Y. We take a point $p \in \dot{T}^*_Y X$, and assume, at $x = \pi(p) \in Y$,

$$N^*_x(\Omega) \neq T^*_x Y. \tag{1.1}$$

$N^*(\Omega)$ denotes the conormal cone of Ω in Y. We denote by ρ and $\tilde{\omega}$ the natural associated maps from $Y \underset{X}{\times} T^*X$ to T^*Y and T^*X, respectively.

Proposition 1.1. *Let $p \in \dot{T}^*_Y X$ and let $F \in Ob(D^+(X))$. Assume* (1.1) *and $SS(F) \subset \tilde{\omega}\rho^{-1}(N^*(\Omega)^a \underset{Y}{\times} \bar{\Omega})$ in a neighbourhood of p. Then there exists a complex $M^\cdot$ of $\boldsymbol{Z}$-modules such that F is isomorphic to $M^\cdot_\Omega = M^\cdot \otimes_{\boldsymbol{Z}} \boldsymbol{Z}_\Omega$ in $D^+(X; p)$.*

Proof. Since $SS(F) \subset \pi^{-1}(Y)$, it follows from Proposition 6.2.1 of [3] that there exists $G \in Ob(D^+(Y))$ such that F is isomorphic to $\boldsymbol{R}j_*G$ in $D^+(X; p)$ (j denotes the embedding of Y into X). By Proposition 4.1.1 of [3] our assumption implies that $SS(G) \subset T^*Y$ is contained in $N^*(\Omega)^a \underset{Y}{\times} \bar{\Omega}$ in a neighbourhood of $x = \pi(p) \in T^*_Y Y$. Thus we have, taking account of (1.1),

i) $\operatorname{supp}(G) \subset \bar{\Omega}$,

ii) $SS(G) \cap N^*(\Omega) \subset T^*_Y Y$.

By Corollary 4.3.3 of [3], at any point $x' \notin \Omega$,

$$G_{x'} = \boldsymbol{R}\Gamma_{\bar{\Omega}}(G)_{x'} = 0.$$

This implies the natural morphism $G_\Omega \to G$ is an isomorphism in $D^+(Y; x)$.