33. *Deficient Cubic Spline Interpolation*

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1. Introduction. Meir and Sharma [6] have studied the problem of interpolation of matching a cubic spline at one intermediate point and deficient cubic spline at two intermediate points between the successive mesh points. For further results in this direction reference may be made to Dikshit and Rana [5], Chatterjee and Dikshit [3] and Rana [7]. Following Schoenberg [8] and de Boor [2], the problem of deficient cubic spline interpolation has been studied by Dikshit and Powar [4]. Corresponding to foregoing Hermite interpolation problem we shall study in this paper a Hermite Birkhoff interpolation by deficient cubic spline. Interesting studies exhibiting sharp convergence properties for such spline interpolant when $f \in C^3$ or $f \in C^4$ have also been made. Our result, in particular includes the results proved in [7].

2. Existence and uniqueness. Let $P: 0=x_0<x_1<\cdots<x_n=1$ denote a partition of $[0, 1]$ with equidistant mesh points so that $h=x_i-x_{i-1}=1/n$. Let P_3 be the set of all real algebraic polynomials of degree not greater than 3. We define the deficient polynomial spline class $S(3, P)$ as

$$S(3,P)=\{s: s \in C^1[0,1],\ s \in P_3 \text{ for each } [x_{i-1}, x_i],\ i=1,2,\cdots,n\}.$$

Throughout g will denote a nondecreasing function on $[0, 1]$ such that

$$g(x+h)-g(x)=H=\int_0^h dg, \qquad x \in [0, 1-h]. \tag{2.1}$$

Setting

$$h^{r+p}A(r,p)=\int_0^h x^r(h-x)^p dg\,; \qquad r, p=0,1,2,3,$$

we observe that as a consequence of (2.1), we have

$$h^{r+p}A(r,p)=\int_{x_{i-1}}^{x_i} (x-x_{i-1})^r(x_i-x)^p dg, \qquad \text{for } i=1,2,\cdots,n.$$

Writing $\theta_i=(x_i+x_{i-1})/2$ for all i, we propose the following:

Problem 2.1. *Given a function $f \in C^1[0,1]$. Does there exist a unique 1-periodic spline $s \in S(3,P)$ which satisfies the interpolatory conditions:*

$$s'(\theta_i)=f'(\theta_i), \qquad i=1,2,\cdots,n, \tag{2.2}$$

$$\int_{x_{i-1}}^{x_i} (f(x)-s(x))dg=0, \qquad i=1,2,\cdots,n? \tag{2.3}$$

Problem 2.2. *For the function f of Problem 2.1, does there exist a*

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