4. *Index and Flow of Weights of Factors of Type III*

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§1. **Introduction.** V. Jones' theory on index of II_1 factors [5] is a major break-through in recent development of the theory of operator algebras. In the type II_1 case, if index is finite, then a factor and its subfactor are known to possess many similar properties (AFD, Property T, etc.). We would like to investigate a similar problem in the type III set-up.

Let $\mathcal{M}$ be a type III factor with a (type III) subfactor $\mathcal{N}$, and let E be a conditional expectation from $\mathcal{M}$ onto $\mathcal{N}$. The notion of index of E was introduced by the second-named author, [6], based on Connes' spatial theory and Haagerup's theory on operator valued weights, [4]. Throughout the article we assume Index $E<\infty$. To check how similar $\mathcal{M}$ and $\mathcal{N}$ are, we will compare the (smooth part of) flow of weights of $\mathcal{M}$ with that of $\mathcal{N}$. Our main theorem shows that each of the two flows is restricted by the other via the Index E $(<\infty)$-information. More precisely, there exists a single flow (X, T_t), and each of the two flows of weights appears as a (at most Index E to one) factor flow of (X, T_t).

In this announcement we will just sketch a proof of the main theorem. Full details and further results will be published elsewhere.

§2. **Notations and the main theorem.** Let E be a conditional expectation from a factor $\mathcal{M}$ onto its subfactor $\mathcal{N}$. We assume that Index E $<\infty$ and $\mathcal{M}$ and $\mathcal{N}$ are of type III. (If one of $\mathcal{M}$ and $\mathcal{N}$ is of type III, then the other is also of type III.) We will denote by $(X_{\mathcal{M}}, T_t^{\mathcal{M}})$ the flow of weights of $\mathcal{M}$ ([3]). The flow of weights can be computed from the associated crossed product $\tilde{\mathcal{M}}=\mathcal{M}\rtimes_\sigma \boldsymbol{R}$ and the dual action $\{\theta_s^{\mathcal{M}}\}_{s\in\boldsymbol{R}}$ on $\tilde{\mathcal{M}}$. (See [3], [10] for details.) More precisely, the center $Z(\tilde{\mathcal{M}})$ is isomorphic to $L^\infty(X_{\mathcal{M}}, d\mu)$, and by restriction the dual action induces the ergodic automorphism group on $Z(\tilde{\mathcal{M}})$. Then, the non-singular ergodic flow $\{T_t^{\mathcal{M}}\}_{t\in\boldsymbol{R}}$ on $X_{\mathcal{M}}$ is related to $\theta_t^{\mathcal{M}}$ via

$$(\theta_t^{\mathcal{M}}(f))(\omega)=f(T_{-t}^{\mathcal{M}}\omega)\,;\quad \omega\in X_{\mathcal{M}},\quad t\in\boldsymbol{R},\quad f\in Z(\tilde{\mathcal{M}})\cong L^\infty(X_{\mathcal{M}}, d\mu).$$

Theorem. *There exists a flow $(X, \{T_t\}_{t\in\boldsymbol{R}})$ satisfying the following:*

(i) *X is isomorphic to $X_{\mathcal{M}}\times\{1, 2, \cdots, m\}$ (resp. $X_{\mathcal{N}}\times\{1, 2, \cdots, n\}$) as a measure space for some positive integer $m, m\leq$ Index E (resp. positive integer $n, n\leq$ Index E),*

(ii) *the projection map $\pi_{\mathcal{M}}$ (resp. $\pi_{\mathcal{N}}$) from X onto $X_{\mathcal{M}}$ (resp. $X_{\mathcal{N}}$) intertwines T_t and $T_t^{\mathcal{M}}$ (resp. T_t and $T_t^{\mathcal{N}}$):*

$$T_t^{\mathcal{M}}\circ\pi_{\mathcal{M}}=\pi_{\mathcal{M}}\circ T_t,\quad T_t^{\mathcal{N}}\circ\pi_{\mathcal{N}}=\pi_{\mathcal{N}}\circ T_t,\quad t\in\boldsymbol{R}.$$