31. On Certain Subclass of Close-to-convex Functions

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Summary. The object of the present paper is to prove a property of functions belonging to the class $\mathcal{R}_n(\alpha)$ which is the subclass of close-to-convex functions of order α in the unit disk.

1. Introduction. Let $\mathcal{A}_n$ denote the class of functions of the form

$$f(z)=z+\sum_{k=n+1}^{\infty} a_k z^k \qquad (n \in \mathcal{N}=\{1,2,3,\cdots\}) \tag{1.1}$$

which are analytic in the unit disk $\mathcal{U}=\{z:|z|<1\}$. A function $f(z)$ belonging to the class $\mathcal{A}_n$ is said to be convex in the unit disk $\mathcal{U}$ if and only if it satisfies

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>0 \qquad (z \in \mathcal{U}). \tag{1.2}$$

Further, a function $f(z)$ in the class $\mathcal{A}_n$ is said to be close-to-convex of order α $(0\leqq\alpha<1)$ in the unit disk $\mathcal{U}$ if there exists a convex function $g(z)$ $\in \mathcal{A}_n$ such that

$$\operatorname{Re}\left\{\frac{f'(z)}{g'(z)}\right\}>\alpha \tag{1.3}$$

for some α $(0\leqq\alpha<1)$ and for all $z \in \mathcal{U}$.

The concept of close-to-convex functions was introduced by Kaplan [2].

A function $f(z)$ belonging to $\mathcal{A}_n$ is said to be in the class $\mathcal{R}_n(\alpha)$ if and only if it satisfies

$$|f'(z)-1|<1-\alpha \tag{1.4}$$

for some α $(0\leqq\alpha<1)$ and for all $z \in \mathcal{U}$. Noting that

$$f(z) \in \mathcal{R}_n(\alpha) \Longrightarrow \operatorname{Re}\{f'(z)\}>\alpha \qquad (z \in \mathcal{U})$$

and $g(z)=z$ is convex in $\mathcal{U}$, we see that $\mathcal{R}_n(\alpha)$ is the subclass of close-to-convex functions of order α in the unit disk $\mathcal{U}$.

Recently, Nunokawa, Fukui, Owa, Saitoh and Sekine [7] have determined the starlikeness bound of functions $f(z)$ in the class $\mathcal{R}_1(\alpha)$.

Let the functions $f(z)$ and $g(z)$ be analytic in the unit disk $\mathcal{U}$. Then the function $f(z)$ is said to be subordinate to $g(z)$ if there exists a function $w(z)$ analytic in the unit disk $\mathcal{U}$, with $w(0)=0$ and $|w(z)|<1$ $(z \in \mathcal{U})$, such that

$$f(z)=g(w(z)) \tag{1.5}$$

for $z \in \mathcal{U}$. We denote this subordination by

$$f(z)\prec g(z). \tag{1.6}$$

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