1. *Lifting of Local Subdifferentiations and Elliptic Boundary Value Problems on Symmetric Domains. I*

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§1. **Introduction.** In [2], C. V. Coffman has shown the existence of non-radial solutions of

(1.1) $$-\Delta u+u=u^{2N+1},\quad u>0 \quad (\text{in } \Omega),\quad u=0 \quad (\text{on } \partial\Omega)$$

on the annulus domain $\Omega=\{a<|x|<a+c\}\subset \boldsymbol{R}^2$ for $N=1,2,\cdots$. Immediately is seen that his method applies to

(1.2) $$-\Delta u=u^p,\quad u>0 \quad (\text{in } \Omega),\quad u=0 \quad (\text{on } \partial\Omega)$$

on the same domain Ω for $1<p<\infty$, and implies the relation

(1.3) $$\sharp\,\{\text{solutions for (1.2)}\}/\mathcal{G}\longrightarrow+\infty$$

as $a\to\infty$, c being fixed, where $\mathcal{G}$ denotes the group of homeomorphisms induced by rotations of independent variables $x\in\Omega$. The proof consists of two parts.

Namely, let $J(v)=\|\nabla v\|_{L^2}/\|v\|_{L^{p+1}}$ and $K_k=\{v\in H_0^1(\Omega)\,|\,T_k v=v,\ v\geqq 0\}\setminus\{0\}$, where T_k denotes the rotation of independent variables $x=\mathrm{r}e^{i\theta}$ for $k=1,2,\cdots$:

$$(T_k v)(\mathrm{r}e^{i\theta})=v(\mathrm{r}e^{i(\theta+(2\pi/k))}).$$

Further, let $K_\infty=\{v\in H_0^1(\Omega)\,|\,v \text{ is radial, } v\geqq 0\}\setminus\{0\}$. Then, first a theorem due to Z. Nahari [8] assures us that each solution $u=u_k$ $(k=1,2,\cdots,\infty)$ of the local variational problem

(1.4) $$\text{To minimize } J(v) \qquad \text{on } v\in K_k$$

satisfies the equation (1.2) by a suitable stretching transformation. Next, the critical values j_k's $(k=1,2,\cdots,\infty)$:

(1.5) $$j_k=\operatorname{Inf}\{J(v)\,|\,v\in K_k\}$$

are separated as $a\to\infty$ after somewhat technical calculations, which guarantees (1.3).

In the manner of Steiner's symmetrization, B. Kawohl has refined the second part of above proof in [6]. That is, it holds that

(1.6) $$m\,|\,n \text{ with } m<n \text{ implies } j_m<j_n \text{ provided that } j_n<j_\infty.$$

Therefore, (1.3) is reduced to showing that for any $m\in N$, $j_m<j_\infty$ follows in the case that a is sufficiently large for each fixed $c>0$.

Still the first part of the above proof depends heavily on the homogenity property of the nonlinear term $f(u)=u^p$ and seems to be impossible to extend into general cases in its original form. However, in the present paper we shall show that generally, solutions of local variational problem

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