21. On Siegel Series for Hermitian Forms

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(Communicated by Shokichi IYANAGA, M. J. A., March 12, 1987)

Let K be an imaginary quadratic number field of discriminant d_K with ring of integers o_K. We let $\Omega_n(K)$ denote the set of hermitian matrices in $M_n(K)$ and put $\Omega_n(o_K)=\Omega_n(K)\cap M_n(o_K)$. An element $H=(h_{ij})\in\Omega_n(K)$ is called *semi-integral* if $h_{kk}\in Z$ and $\sqrt{d_K}\,h_{ij}\in o_K$ $(i\neq j)$. Denote by $\Lambda_n(K)$ the set of semi-integral matrices in $\Omega_n(K)$. For an element H in $\Lambda_n(K)$, we define a *singular series* by

$$b(s, H)=\sum_R \nu(R)^{-s}\exp[2\pi i tr(HR)], \qquad s\in C,$$

where R runs over all hermitian matrices mod $\Omega_n(o_K)$ and $\nu(R)$ denotes the determinant of the denominator of R (cf. [1]). If Re$(s)>2n$, then the series is absolutely convergent. In the case of quadratic forms, this series was studied by Siegel [6], Kaufhold [2], Shimura [5] and Kitaoka [3]. The purpose of this note is to give an explicit formula for the series $b(s, H)$ under a certain condition.

In the rest of this note, we assume that the class number of K is 1 and $n=2$. For each hermitian matrix R in $M_2(K)$, we have a unique decomposition $R\equiv\sum R_p$ mod $\Omega_2(o_K)$ where R_p is a hermitian matrix in $M_2(K)$ such that $\nu(R_p)$ is a power of rational prime p. Therefore we have a decomposition

$$b(s, H)=\prod_p b_p(s, H),$$
$$b_p(s, H)=\sum_{R_p}\nu(R_p)^{-s}\exp[2\pi i tr(HR_p)],$$

where R_p runs over all hermitian matrices mod $\Omega_2(o_K)$ such that $\nu(R_p)$ is a power of rational prime p. Thus our problem is reduced to finding a formula for $b_p(s, H)$. The series $b_p(s, H)$ was studied by Shimura in [5] under the general situation and is called *Siegel series associated with H*.

We fix a rational prime p. For each non-zero matrix H in $\Lambda_2(K)$, and put $d_1(H)=\max\{m\in Z \mid m^{-1}H\in\Lambda_2(K)\}$ and $p^{\alpha(H)}\| d_1(H)$. When H is non-singular we determine the integers $a(H)$, $d(H)$ and $d_p(H)$ by $p^{a(H)}\| d(H)=|\sqrt{d_K}H|$ (the determinant of $\sqrt{d_K}H$), $d(H)=p^{a(H)}d_p(H)$. We note that $a(H)\geqq 2\alpha(H)\geqq 0$.

The first result can be stated as follows.

Theorem 1. *Let H be a non-zero matrix in $\Lambda_2(K)$ and $\chi(\cdot)$ denote the Kronecker symbol of K.*

(1) *If $|H|\neq 0$, then*

$$b_p(s, H)=(1-p^{-s})(1-\chi(p)p^{1-s})F_p(s, H),$$

where