96. Propagation of Singularities for Microdifferential Equations with Multiple Self-tangential Involutory Characteristics

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§1. **Introduction.** We study a class of microdifferential equations with multiple involutory characteristics. Explicitly, let M be a real analytic manifold of dimension n with a complex neighborhood X and let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$ module defined in a neighborhood of $\rho_0 \in T^*_M X \backslash M$. (See M. Sato *et al.* [4] and P. Schapira [5] for $\mathcal{E}_X$.) We assume that the characteristic variety of $\mathfrak{M}$ is written in a neighborhood of ρ_0 as

(1) $$\mathrm{ch}(\mathfrak{M}) = \{\rho \in T^*X\,;\, p_1(\rho)\cdot p_2(\rho)\cdots p_l(\rho)=0\}$$

by homogeneous holomorphic functions $p_1, \cdots, p_{l-1}$ and p_l defined in a neighborhood of ρ_0. Here $p_1, \cdots, p_{l-1}$ and p_l satisfy the following conditions (2), (3) and (4).

(2) $p_1, \cdots, p_{l-1}$ and p_l are real valued on $T^*_M X$.

We set $S_j = \{\rho \in T^*_M X\,;\, p_j(\rho)=0\}$ $(1 \le j \le l)$ and assume

(3) S_j's are regular (non-radical) non-singular hypersurfaces and $\Sigma = \bigcap_{1\le j\le l} S_j$ is a regular involutory submanifold of $T^*_M X$ of codimension d.

(4) S_i and S_j are tangent to each other of order $k_0 (\ge 1)$ on Σ in case $i \ne j$. This implies that the jets of S_i and S_j coincide up to order k_0 and that S_i and S_j intersect only on Σ if $i \ne j$.

The above class of equations is studied by N. Dencker [1] in C^∞ case and we study the analytic case under somewhat weaker conditions. The author emphasizes here that we pose no assumption on the multiplicities of the equations and that only the geometry of the characteristic varieties is concerned if we employ the theory of microlocal study of sheaves due to M. Kashiwara and P. Schapira [3]. See also N. Tose [9], [10] and [12] for related results about propagation of singularities for systems with involutory characteristics.

§2. **Notation.** To state the results, we give some prerequisites about 2-microfunctions.

Let Λ be a complexifications of Σ in T^*X. Then $\tilde{\Sigma}$ denotes the union of all bicharacteristic leaves of Λ issued from Σ. M. Kashiwara introduced the sheaf $\mathcal{C}^2_\Sigma$ of 2-microfunctions along Σ on $T^*_\Sigma \tilde{\Sigma}$. By $\mathcal{C}^2_\Sigma$, we can study the properties of microfunctions on Σ precisely. Actually, we have exact sequences

(5) $$0 \longrightarrow \mathcal{C}_{\tilde{\Sigma}}|_\Sigma \longrightarrow \mathcal{B}^2_\Sigma \longrightarrow \pi_{\Sigma *}(\mathcal{C}^2_\Sigma|_{T^*_\Sigma \tilde{\Sigma}\backslash\Sigma}) \longrightarrow 0 \qquad (\pi_\Sigma : T^*_\Sigma\tilde{\Sigma}\backslash\Sigma \longrightarrow \Sigma)$$