

93. Differential Invariants of Superwebs

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§ 1. Introduction. Let M be a superdomain of $2|2$ -dimensional super Euclidean space $E^{2|2}$. We consider on M a natural analogy of web (a system of 3-families of curves on a surface), which will be called a superweb.

Blaschke and Dubourdieu ([2]) solved the local equivalence problem of webs by constructing natural torsion free connections. This result implies that all the differential invariants of a web are generated by the curvature.

In this note we shall solve the local equivalence problem of superwebs. We also clarify the relations between superwebs and webs. Details will be published elsewhere.

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§ 2. Distributions on superdomains. For the sake of brevity, we adopt the Batchelor's formalism of supermanifolds ([1]). In this paragraph, we recall requisites for the subsequent arguments.

A real Grassmann algebra Λ is fixed as the coefficient ring of the theory. It is assumed that the number of odd generators of Λ is sufficiently large (by converting Λ , if needed. cf. [1]). The $m|n$ -dimensional super Euclidean space $E^{m|n}$ is the direct sum of m copies of Λ_0 and n copies of Λ_1 with the coarse topology. A coarse open set M of $E^{m|n}$ is called a *superdomain*.

A *distribution* \mathcal{D} on M of codimension $r|s$ is locally given by Pfaffian equations $\varphi^1 = \cdots = \varphi^r = \psi^1 = \cdots = \psi^s = 0$, which will be called a *system of local equations* for \mathcal{D} . Here φ^i 's are even and ψ^j 's are odd 1-forms that are independent over the superalgebra of the supersmooth functions. We call \mathcal{D} a *foliation* if it is completely integrable.

§ 3. Superwebs. Let M be a superdomain of dimension $2|2$ and $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$ be foliations on M of codimension $1|1$. Let $\varphi_i = \psi_i = 0$ be a system of local equations for $\mathcal{D}_i, i=1, 2, 3$. We call a triple $\mathcal{W} = \{\mathcal{D}_i\}$ a *superweb* on M when $\varphi_i, \varphi_j, \psi_i, \psi_j$ form a coframe field if $i \neq j$. Two superwebs $\{\mathcal{D}_i\}$ and $\{\mathcal{D}'_i\}$ are called *equivalent* if there is a superdiffeomorphism f such that $f_* (\mathcal{D}_i) = \mathcal{D}'_i, i=1, 2, 3$. Such a superdiffeomorphism is called an *equivalence* of the superwebs.

§ 4. $GL(1|1, \Lambda)$ -structures. For a superweb $\mathcal{W} = \{\mathcal{D}_i\}$, we can choose, by normalizing, systems of local equations for \mathcal{D}_i 's satisfying $\varphi_3 = \varphi_1 + \varphi_2$ and $\psi_3 = \psi_1 + \psi_2$. Such systems will be called *normal*. If $\varphi_i = \psi_i = 0$ and