71. On a Class of Nonhyperbolic Microdifferential Equations with Involutory Double Characteristics

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(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 14, 1987)

§1. **Introduction.** In this note, we study a class of microdifferential equations with involutory double characteristics. Explicitly, let M be a real analytic manifold of dimension n (≥ 4) with a complexification X. We consider a microdifferential equation defined in a neighborhood of $\rho_0 \in T^*_M X \setminus M$

(1) $$Pu=\{(P_1+\sqrt{-1}P_2)P_3+Q\}u=0.$$

Here we set $p_j=\sigma(P_j)(1\leq j\leq 3)$ and assume the following conditions.

(2) $\operatorname{ord}(P_1)=\operatorname{ord}(P_2)=m_1$, $\operatorname{ord}(P_3)=m_2$ and $\operatorname{ord}(Q)=m_1+m_2-1$.

(3) p_1, p_2 and p_3 are real valued on $T^*_M X$.

(4) $p_j(\rho_0)=0 \quad (1\leq j\leq 3)$.

(5) dp_1, dp_2, dp_3 and the canonical 1-form ω of $T^*_M X$ are linearly independent at ρ_0.

(6) $\{p_i, p_j\}=0$ if $p_i=p_j=0$ $(1\leq i, j\leq 3)$ where $\{\cdot, \cdot\}$ denotes Poisson bracket on $T^*_M X$.

By Sato *et al.* [4], the structure of microdifferential equation (1) is completely studied outside the regular involutory submanifold

(7) $$\Sigma=\{\rho\in T^*_M X\,;\ p_1(\rho)=p_2(\rho)=p_3(\rho)=0\}.$$

Thus, we interest ourselves in studying the structure of solutions on Σ. By employing the theory of 2-microlocalization due to M. Kashiwara and Y. Laurent (see [1], [3]), we show a result about the propagation of 2-microlocal singularities as a byproduct of N. Tose [6]. More precisely, we see the equation (1) is 2-microlocally equivalent to $(D_1+\sqrt{-1}D_2)u=0$ or $D_3u=0$ or $u=0$.

§2. **Preliminary.** 2.1. *2-microdifferential operators.* Let X be an open subset in C^{n+d} and let T^*X be its cotangent bundle. We take a coordinate of X as (w, z) with $w\in C^n$ and $z\in C^d$. Then $\rho=(w, z\,;\theta dw+\zeta dz)$ denotes a point of T^*X with $\theta\in C^n$ and $\zeta\in C^d$. For microdifferential operators, see M. Sato *et al.* [4] and P. Schapira [5].

Hereafter in §2.1, Λ is the regular involutory submanifold in $T^*X\setminus X$: $\Lambda=\{(w, z\,;\theta, \zeta)\,;\zeta=0\}$. We identify Λ with a submanifold of $\Lambda\times\Lambda$ through the embedding $T^*X\simeq T^*_X(X\times X)\subsetneq T^*(X\times X)$. By definition, $\tilde{\Lambda}$ is the union of bicharacteristic leaves of $\Lambda\times\Lambda$ issued from Λ. We take a coordinate of $T^*_\Lambda\tilde{\Lambda}$ as $(w, z\,;\theta\,;z^*)$ with $(w, z\,;\theta)\in\Lambda$ and $z^*\in C^d$.

$T^*{}_\Lambda\tilde{\Lambda}$ is endowed with the sheaf $\mathcal{E}^{2,\infty}_\Lambda$ of 2-microdifferential operators of infinite order constructed in Y. Laurent [3].

Definition 1. For an open subset U of $T^*_\Lambda\tilde{\Lambda}$, a formal sum