67. Mixed Problems for Quasi-Linear Symmetric Hyperbolic Systems

By Shuichi KAWASHIMA*), Taku YANAGISAWA,**) and Yasushi SHIZUTA**)

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1. Introduction. Our primary interest in this note is the mixed problem for the first order quasi-linear hyperbolic systems with characteristic boundary. The case where the boundary matrix is nonsingular has been investigated by several authors, but we do not enter into detail here. (See [5] and the references therein.) The characteristic boundary value problem was treated by Tsuji [6], Majda-Osher [1], Ohkubo [2] and Rauch [4]. Recently, Ohkubo [3] gave an improved version of his sufficient condition for the full regularity of solutions to the linear mixed problem and established a local existence theorem for the quasi-linear mixed problem. Our purpose in this paper is to present another method for solving the quasi-linear mixed problem. To do this, we formulate a new sufficient condition which seems to be somewhat weaker than Ohkubo's one.

2. Assumptions and main result. Let Ω be a bounded domain in $\boldsymbol{R}^n$ with smooth, compact boundary $\partial\Omega$. We study the following mixed problem.

(1)$_1$ $\quad A^0(t,x,u)u_t+\sum_{j=1}^{n} A^j(t,x,u)u_{x_j}=f(t,x,u) \quad \text{in } [0,T]\times\Omega,$

(1)$_2$ $\quad M(x)u=0 \quad \text{on } [0,T]\times\partial\Omega,$

(1)$_3$ $\quad u(0,x)=u_0(x) \quad \text{for } x\in\Omega.$

Here the unknown $u=u(t,x)$ is a vector-valued function with m components and takes values in a convex open set $\mathcal{O}\subset\boldsymbol{R}^m$, A^0 and A^j, $j=1,\cdots,n$, are smoothly varying real $m\times m$ matrices defined on $[0,T]\times\bar{\Omega}\times\mathcal{O}$, and f is a smooth function on $[0,T]\times\bar{\Omega}\times\mathcal{O}$ with values in $\boldsymbol{R}^m$. M is a real $r\times m$ matrix ($r<m$) depending smoothly on $x\in\partial\Omega$. It is assumed that M is of full rank for $x\in\partial\Omega$.

Condition 1. $A^0(t,x,u)$ is real symmetric and positive definite for $(t,x,u)\in[0,T]\times\bar{\Omega}\times\mathcal{O}$. $A^j(t,x,u)$, $j=1,\cdots,n$, are real symmetric for $(t,x,u)\in[0,T]\times\bar{\Omega}\times\mathcal{O}$.

We write $\partial_j=\partial/\partial x_j$, $j=1,\cdots,n$, and put $\partial_x=(\partial_1,\cdots,\partial_n)$. For a first order differential operator $A(t,x,u;\partial_x)=\sum_{j=1}^n A^j(t,x,u)\partial_j$, we denote its symbol by $A(t,x,u;\xi)=\sum_{j=1}^n A^j(t,x,u)\xi_j$, where $\xi=(\xi_1,\cdots,\xi_n)\in\boldsymbol{R}^n$. Let $\nu(x)$ be the unit outward normal to $\partial\Omega$ at x. The null space of $M(x)$ is the boundary subspace and is denoted by $\mathcal{N}(M(x))$.

*) Department of Applied Science, Faculty of Engineering 36, Kyushu University, Fukuoka 812.

**) Department of Mathematics, Nara Women's University, Nara 630.