49. A Result on the Scattering Theory for First Order Systems with Long-range Perturbations

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In this report we treat the following differential equation for $C^\omega$-valued function:

$$D_t u = \Lambda u,$$

where $D_t = (1/i)(\partial / \partial t)$ and

$$\Lambda = E(x)^{-1/2} \sum_{j=1}^m A_j D_j E(x)^{-1/2},$$

$A_j$'s are $m \times m$ constant hermitian matrices, and $E(x)$ is a continuous $m \times m$ hermitian matrix valued function with

$$0 < c_1 I \leq E(x) \leq c_2 I$$

for some constants $c_1$ and $c_2$. $\Lambda$ can be extended to a self-adjoint operator on $\mathcal{H} = L^2(\mathbb{R}^n)$. If we substitute $E(x)$ with $I$ in (1), we have a differential operator of constant coefficients:

$$\Lambda^0 = \sum_{j=1}^m A_j D_j.$$

$\Lambda^0$ can also be extended to a self-adjoint operator on $\mathcal{H}$, and $\Lambda$ is regarded as a perturbed operator of $\Lambda^0$. The main result which we shall report here is the existence theorem of the wave operator between $\Lambda^0$ and $\Lambda$. We consider the case that the perturbation is long-range. More precisely we assume that

**Assumption (E).**

1) $E(x) \in C^\omega(\mathbb{R}^n)$.
2) $|\partial^\alpha E(x) - I| \leq (1 + |x|)^{-\delta - |\alpha|}$ for $\delta > 0$ and $|\alpha| \geq 0$.

The operator $W_\alpha$ is called the wave operator if the limit

$$W_\alpha u = \lim_{\epsilon \to 0} e^{\epsilon u} e^{it \Lambda^0 u} \quad (u \in \mathcal{H}_\alpha(\Lambda^0))$$

exists. In the case of the short-range ($\delta > 1$) it is already known that, for wide class of $\Lambda^0$, $W_\alpha$ exists and is complete (see for example [3]). But it does not exist generally when the perturbation is long-range ($0 < \delta \leq 1$). Then we should consider the modified wave operator. The fundamental problems of the theory of long-range perturbation are the existence and completeness of the modified wave operator. However few works have been treated related to the spectral theory of systems with long-range perturbations. There are only the works related to the limiting absorption principle ([3], [4]). Then unlike the case of the short-range the existence theorem is the first step of this theory.

On $\Lambda^0$ we assume the following. We put

$$\Lambda^0(\xi) = \sum_{j=1}^m A_j \xi_j \quad \text{(symbol of $\Lambda^0$)}.$$