6. *Variations of Pseudoconvex Domains in the Complex Manifold*

By Hiroshi YAMAGUCHI
Faculty of Educations, University of Shiga

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Introduction. In the n-dimensional complex vector space C^n with standard norm $\|z\|^2=|z_1|^2+\cdots+|z_n|^2$ for $z=(z_1,\cdots,z_n)\in C^n$, let D be a relatively compact domain of C^n with smooth boundary. Given $\zeta\in D$, D carries the Green's function $G(z)$ with pole at ζ for the Laplace equation $\Delta G=(\partial^2/\partial z_1\partial\bar{z}_1+\cdots+\partial^2/\partial z_n\partial\bar{z}_n)G=0$. The function $G(z)$ is expressed in the form

$$G(z)=\begin{cases}-\log|z-\zeta|+\lambda+H(z) & (n=1)\\ \|z-\zeta\|^{-2n+2}+\lambda+H(z) & (n\geqq 2)\end{cases}$$

where λ is a constant, $H(z)$ is harmonic in D and $H(\zeta)=0$. The constant term λ is called the Robin constant for $(D,\{\zeta\})$. When D varies in C^n with parameter t, so does λ with t. This is realized as follows: Let B be a domain of the t-complex plane containing the origin O. We let correspond to each $t\in B$ a relatively compact domain $D(t)$ of C^n with smooth boundary such that $D(t)\ni\zeta$ for all $t\in B$ and $D(O)=D$, and denote by $\lambda(t)$ the Robin constant for $(D(t),\{\zeta\})$. Consequently, $\lambda(t)$ defines a real-valued function on B. In [6] we showed

Theorem 1. *If the set $\tilde{D}=\{(t,z)\in B\times C^n\,|\,z\in D(t)\}$ is a pseudoconvex domain in $B\times C^n$, then $\lambda(t)$ is a superharmonic function on B.*

In this note we extend Theorem 1 to the case when $D(t)$ are domains in a complex manifold M.

1. Let M be a (compact or non-compact) connected complex manifold of dimension n. In this note we always assume that $n\geqq 2$, for we studied in [5] the case of $n=1$. Let $ds^2=\sum_{\alpha,\beta=1}^n g_{\alpha\beta}dz_\alpha\otimes d\bar{z}_\beta$ be a Hermitian metric on M. For notations we follow [3]. We put

$$\omega=i\sum_{\alpha,\beta=1}^n g_{\alpha\beta}dz_\alpha\wedge d\bar{z}_\beta,\qquad \omega^n=(i)^n n!\,g(z)dz_1\wedge d\bar{z}_1\wedge\cdots\wedge dz_n\wedge d\bar{z}_n,$$

$$\Delta=-(*\partial*\bar{\partial}+*\bar{\partial}*\partial)=-2\left\{\sum_{\alpha,\beta=1}^n g^{\alpha\beta}\frac{\partial^2}{\partial\bar{z}_\alpha\partial z_\beta}+\mathrm{Re}\sum_{\alpha,\beta=1}^n\frac{1}{g}\frac{\partial(gg^{\alpha\beta})}{\partial\bar{z}_\alpha}\frac{\partial}{\partial z_\beta}\right\},$$

where $i^2=-1$, $g(z)=\det(g_{\alpha\beta}(z))$ and $(g^{\alpha\beta}(z))=(g_{\alpha\beta}(z))^{-1}$. If a function u defined in a domain of M is of class C^2 and satisfies $\Delta u=0$, then u is said to be harmonic. For $\zeta\in M$ and a neighborhood U of ζ, we denote by $E(\zeta,U,ds^2)$ the set of all elementary solutions $E(\zeta,z)$ for $\Delta E(\zeta,z)=0$ on $U\times U$ except for the diagonal set (see K. Kodaira [2], p. 612).

In what follows, if M is compact, then we assume $D\neq M$. Moreover, we suppose $\zeta\in D$ and $E(\zeta,z)\in E(\zeta,U,ds^2)$.

First, consider the case where D is a relatively compact domain of M