103. *Mixed Hodge Modules*

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Introduction. We define $MHM(X, k)^{(g)}$ the categories of (*geometric*) *mixed Hodge Modules* in the algebraic case, and prove the stability by subquotients, vanishing cycle functors, direct images, pull-backs (and external products). In this note, X, Y are smooth algebraic varieties (assumed always separated) over $\boldsymbol{C}$, and $\mathcal{D}_X$ is the sheaf of algebraic differential operators; all the $\mathcal{D}_X$-Modules are assumed quasi-coherent, and the holonomic Modules regular.

§1. Definitions and main results.

1.1. Let k be a subfield of $\boldsymbol{R}$. Let $MF_h(\mathcal{D}_X, k)$ be the category of filtered holonomic $\mathcal{D}_X$-Modules (M, F) with k-structure given by $DR(M) \simeq \boldsymbol{C} \otimes K$ for $K \in \mathrm{Perv}(k_X)$, $MH_Z(X, k, n)^p$ the category of (algebraic) *polarizable Hodge Modules* of weight n with strict support Z, and $MH(X, k, n)^p := \oplus MH_Z(X, k, n)^p$ (cf. [4, 5]). $MHW(X, k)^p$ is the category of the objects of $MF_h(\mathcal{D}_X, k)$ with a finite filtration W such that $\mathrm{Gr}_i^W \in MH(X, k, i)^p$ for any i.

1.2. Let g be a function on X. Then by definition

$$\psi_g(M, F, K) = (\oplus_{-1 \le \alpha < 0} (Gr_\alpha^V \tilde{M}, F[1]), \psi_g K[-1]),$$
$$\phi_{g,1}(M, F, K) = ((\mathrm{Gr}_0^V \tilde{M}, F), \phi_{g,1} K[-1]),$$

for $(M, F, K, W) \in MHW(X, k)^p$, where $(\tilde{M}, F) = i_{g*}(M, F)$ with i_g the immersion by graph, and V is the filtration of Malgrange-Kashiwara (cf. [loc. cit]). Let L be the filtration defined by $L_i \psi_g = \psi_g W_{i+1}$ and $L_i \phi_{g,1} = \phi_{g,1} W_i$. We say that the vanishing cycle functors ψ_g and $\phi_{g,1}$ are *well-defined* for $(M, F, K, W) \in MHW(X, k)^p$, if the following conditions are satisfied (compare to [6]):

(1.2.1) (F, W, V) are compatible filtrations (cf. [5]) of $\tilde{M}$,

(1.2.2) the monodromy filtration W of ψ_g and $\phi_{g,1}$ relative to L exists (cf. [3]),

(1.2.3) $\mathrm{can}(W_i \psi_{g,1}) \subset W_i \phi_{g,1}$ and $\mathrm{Var}(W_i \phi_{g,1}) \subset W_{i-2} \psi_{g,1}(-1)$,

(1.2.4) (F, W, L) are compatible filtrations of ψ_g and $\phi_{g,1}$,

(1.2.5) $(\psi_g(M, F, K), W)$, $(\phi_{g,1}(M, F, K), W) \in MHW(X, k)^p$.

(As is pointed out by Kashiwara, (1.2.3–5) follows from the other conditions.)

1.3. Let $i : U \to X$ be an open immersion such that $X \backslash U$ is a divisor. Let $E = (M, F, K, W) \in MHW(U, k)^p$. Then $E' = (M', F, K', W) \in MHW(X, k)^p$

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