94. Representations of a Solvable Lie Group on $\bar{\partial}_b$ Cohomology Spaces

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Let $(\mathfrak{g}, j, \omega)$ be a normal j-algebra introduced by Pyatetskii-Shapiro [5] (see below). We denote by G the connected and simply connected Lie group with Lie algebra $\mathfrak{g}$. The aim of this note is to give, relating its construction to a certain geometric structure, a unitary representation of G in which every irreducible (up to a set of Plancherel measure zero) occurs with multiplicity one.

1. A triplet $(\mathfrak{g}, j, \omega)$ of a completely solvable Lie algebra $\mathfrak{g}$, a linear operator j on $\mathfrak{g}$ such that $j^2=-1_{\mathfrak{g}}$ and $\omega \in \mathfrak{g}^*$ is termed a *normal j-algebra* if (i) the Nijenhuis tensor for j vanishes, (ii) $\langle x, y\rangle := \omega([x, jy])$ defines an inner product on $\mathfrak{g}$ relative to which j is an orthogonal transformation. Let $G=\exp \mathfrak{g}$, the connected and simply connected Lie group corresponding to $\mathfrak{g}$. As is well-known, there is a Siegel domain D of type II on which G acts simply transitively by affine automorphisms. Denote by $S(D)$ the Šilov boundary of D. Then, $S(D)$ is diffeomorphic to a nilpotent (at most 2-step) normal subgroup $N(D)$ of G. Moreover, G is written as a semidirect product $G=N(D)\rtimes G(0)$ with a closed subgroup $G(0)$ of G. We assume throughout this note that D does not reduce to a tube domain. In this case, $N(D)$ is a 2-step nilpotent Lie group and $S(D)$ has a natural CR structure. So, the tangential Cauchy-Riemann operator $\bar{\partial}_b$ is defined and we have $\bar{\partial}_b \circ \bar{\partial}_b=0$.

By Rossi-Vergne [7], the unitary representations of $N(D)$ defined by translations on the square integrable $\bar{\partial}_b$ cohomology spaces H^q $(q=0, 1, \cdots)$ on $S(D)$ contain almost every irreducible of $N(D)$. We will define unitary representations of G on H^q $(q=0, 1, \cdots)$ such that their restrictions to $N(D)$ coincide with those of [7]. We remark that there is no G-invariant Riemannian metric on $S(D)$, so the usual geometric method is not directly applicable.

2. It is known that $\mathfrak{g}$ is written as an orthogonal direct sum (relative to $\langle\cdot, \cdot\rangle$) $\mathfrak{g}=\mathfrak{g}(0)\oplus\mathfrak{g}(1/2)\oplus\mathfrak{g}(1)$ with $[\mathfrak{g}(k), \mathfrak{g}(m)]\subset\mathfrak{g}(k+m)$, where we understand $\mathfrak{g}(k)=\{0\}$ for $k>1$. Then, $\mathfrak{g}(0)=\mathrm{Lie}\, G(0)$ and we have $\mathfrak{n}(D) := \mathrm{Lie}\, N(D)$ $=\mathfrak{g}(1/2)+\mathfrak{g}(1)$. We put $V=\mathfrak{g}(1/2)$. Then V is j-invariant, so $\dim V>0$ is even. We denote by Ξ the set of all $\lambda \in \mathfrak{g}(1)^*$ such that the skew-symmetric bilinear form $\lambda([x, y])$ on V is non-degenerate. Ξ is an open dense subset of $\mathfrak{g}(1)^*$. Let J be a Borel mapping with values in real linear operators on V such that for each $\lambda \in \Xi$, (i) $J(\lambda)$ is a complex structure on V satisfying