68. 2-nd Microlocalization and Conical Refraction

By Nobuyuki TOSE
Department of Mathematics, Faculty of Science, Ehime University

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§1. **Introduction.** The phenomenon of conical refraction has long been observed by physicists. It is attributed to the non-uniformity of multiplicities to Maxwell equation in the crystal and studied in the framework of Microlocal Analysis by Melrose-Uhlmann [8] and P. Laubin [5], [6].

We employ the theory of 2-microlocalization developed by M. Kashiwara and Y. Laurent (see [2], [4]) and gain a new insight about the phenomenon.

Explicitly, let P be a microdifferential operator defined in a neighborhood of $\rho_0 \in \sqrt{-1}\mathring{T}^*\boldsymbol{R}^n$, which satisfies the following conditions.

(1) P has a real principal symbol p.

Let $\Sigma_1=\{\rho \in \sqrt{-1}T^*\boldsymbol{R}^n\,;\, p(\rho)=0\}$ and $\Sigma_2=\{\rho \in \Sigma_1\,;\, dp(\rho)=0\}$.

(2) Σ_2 is a regular involutory submanifold in $\sqrt{-1}T^*\boldsymbol{R}^n$ through ρ_0 of codimension $d\geqq 3$.

(3) Hess $p(\rho)$ has rank d with positivity 1 if $\rho \in \Sigma_2$.

Moreover we assume

(4) P has regular singularities along Σ_2^C in the sense of Kashiwara-Oshima [3], where Σ_2^C denotes a complexification of Σ_2 in $T^*\boldsymbol{C}^n$.

Our main interest is the propagation of singularities on Σ_2 for the equation $Pu=0$, which can be transformed by a quantized contact transformation into

$$P_0u=\Big(D_1^2-\sum_{i,j=2}^{d} A^{ij}(x, D)D_iD_j+(\text{lower})\Big)u=0 \tag{5}$$

defined in a neighborhood of $\rho_1=(0, \sqrt{-1}dx_n)$. Here A^{ij} are of order 0 with $(\sigma(A^{ij}))$ positive definite. We remark that in this case $\Sigma_2=\{(x, \sqrt{-1}\xi)\,;\, \xi_1=\cdots=\xi_d=0\}$ and that P_0 has regular singularities along Σ_2^C.

We study (5) 2-microlocally along Σ_2. After transforming (5) by a quantized homogeneous bicanonical transformation, which is wider than quantized contact transformations, we give the canonical form of (5) as $D_1u=0$. Then we can easily obtain a theorem about the propagation of 2-microlocal singularities.

§2. **Notation.** Let X be a complex manifold and Λ be a regular involutory submanifold of T^*X. Λ is embedded naturally into $\Lambda\times\Lambda$. $\tilde{\Lambda}$ denotes the union of all bicharacteristics of $\Lambda\times\Lambda$ that pass through Λ. $\mathcal{E}_\Lambda^{2,\infty}$ is the sheaf on $T_\Lambda^*\tilde{\Lambda}$ of 2-microdifferential operators constructed by Y. Laurent [4].

Let M be a real analytic manifold whose complexification is X. Σ denotes a regular involutory submanifold of T_M^*X, whose complexification