66. Canonical Bundles of Compact Complex Surfaces containing Global Spherical Shells

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The purpose of this note is to determine the numerical class of the canonical bundle of a compact complex surface S containing a global spherical shell (GSS for short). Kato [3] first introduced the notion of GSS, and Nakamura [4] classified all surfaces containing GSS's. Nakamura also computed the intersection matrix of curves on such a surface S. In this note, we shall write down the numerical class of the canonical bundle K_S in terms of the intersection matrix. This is one of the problems raised by Dloussky [1].

Details of this note will be published elsewhere.

Notation. Let $A=A_1+\cdots+A_n$ be a linear chain of curves on a surface. Then Zykel(A) denotes $(a_1, \cdots, a_n)$, where the self-intersection number of A_i is $-a_i$.

§1. Let S be a compact complex surface containing a GSS. For the definition of GSS, we refer to Kato [3]. We assume that S has no exceptional curve of the first kind and that the second Betti number $b_2(S)$ is positive. Then, using results of Enoki [2] and Nakamura [4], [5], the surface S is classified as follows.

Theorem 1.1. *S is one of the following surfaces*: (i) *a hyperbolic Inoue surface*, (ii) *a half Inoue surface*, (iii) *a parabolic Inoue surface*, (iv) *an exceptional compactification $S_{n,\beta,t}$ of an affine line bundle on an elliptic curve*, (v) *a* (CB)-*surface.*

The surfaces in the classes (i), (ii) and (iii) are defined in [5], one in the class (iv) is defined in [2], and a (CB)-surface S is defined as follows: S has only finite number of curves, which are rational curves and constitute a single cycle with linear branches sprouting from the cycle.

The surfaces in the classes (i), (ii), (iii) and (iv) have been well studied, and the canonical bundles of them are easily obtained. So, from now on, we let S be a (CB)-surface. The intersection matrix of curves on S was calculated by Nakamura [4] as follows.

Theorem 1.2. *Let C be the set of all curves on S. Then C is decomposed as $C=\sum_{k=1}^{m}(C_k+D_k)$, where*

(i) *(C_k+D_k) has the type $(p_1, q_1, p_2, q_2, \cdots, q_{n-1}, p_n)$*, i.e., *the self-intersection number of components of C_k and D_k are one of the following*:

(1) *if $p_1\geqq 3$, then*