62. *Proof of Masser's Conjecture on the Algebraic Independence of Values of Liouville Series*

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(Communicated by Shokichi IYANAGA, M. J. A., June 10, 1986)

Let $f(z)=\sum_{k=1}^{\infty} z^{k!}$. Then $f(z)$ converges in $|z|<1$. If α is an algebraic number with $0<|\alpha|<1$, then $f(\alpha)$ is a transcendental number. Masser conjectured that if $\alpha_1, \cdots, \alpha_n$ are algebraic numbers with $0<|\alpha_i|<1$ $(1\leqq i\leqq n)$ and no α_i/α_j $(1\leqq i<j\leqq n)$ is a root of unity, then $f(\alpha_1), \cdots, f(\alpha_n)$ are algebraically independent. In [2], the author proved the p-adic analogue of the conjecture, and in [3], settled the conjecture for $n=3$ in complex case. In this paper we shall prove the following theorem by using Evertse's Theorem 1 in [1].

Theorem. *Suppose $\alpha_1, \cdots, \alpha_n$ are algebraic numbers with $0<|\alpha_i|<1$ $(1\leqq i\leqq n)$ and no α_i/α_j $(1\leqq i<j\leqq n)$ is a root of unity. Then $f^{(l)}(\alpha_i)$ $(1\leqq i\leqq n, 0\leqq l)$ are algebraically independent, where $f^{(l)}(z)$ denotes the l-th derivative of $f(z)$.*

In what follows, K will denote an algebraic number field including $\alpha_1, \cdots, \alpha_n$. By a prime on K we mean an equivalence class of non-trivial valuations on K. We denote the set of all primes on K by S_K and the set of all infinite primes on K by S_∞. For every prime v on K lying above a prime p on $\boldsymbol{Q}$, we choose a valuation $\|\cdot\|_v$ such that

$$\|\alpha\|_v=|\alpha|_p^{[K_v:\,\boldsymbol{Q}_p]} \qquad \text{for all } \alpha\in\boldsymbol{Q}.$$

Then we have the product formula:

$$\prod_{v\in S_K}\|\alpha\|_v=1 \qquad \text{for all } \alpha\in K,\ \alpha\neq 0.$$

For $X=(x_0:x_1:\cdots:x_n)\in P^n(K)$, put

$$H_K(X)=H(X)=\prod_{v\in S_K}\max(\|x_0\|_v, \|x_1\|_v, \cdots, \|x_n\|_v).$$

By the product formula, this height is well-defined. Put

$$h_K(\alpha)=h(\alpha)=H(1:\alpha) \qquad \text{for } \alpha\in K.$$

Then so-called fundamental inequality holds,

$$-\log h(\alpha)\leqq\sum_{v\in S}\log\|\alpha\|_v\leqq\log h(\alpha) \qquad \text{for } \alpha\in K,\ \alpha\neq 0,$$

where S is any set of primes on K.

Let S be a finite set of primes on K, enclosing S_∞, and c, d be constants with $c>0$, $d\geqq 0$. A projective point $X\in P^n(K)$ is called (c, d, S)-*admissible* if its homogeneous coordinates $x_0, x_1, \cdots, x_n$ can be chosen such that

(i) all x_k are S-integers, i.e. $\|x_k\|_v\leqq 1$ if $v\notin S$

and

(ii) $\prod_{v\in S}\prod_{k=0}^{n}\|x_k\|_v\leqq c\cdot H(X)^d.$