57. *On Artinian Modules*

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Introduction. Let R be a Noetherian ring, I an ideal of R and A an Artinian R-module. Matlis [3] has defined the notions of A-cosequence and of I-dimension of A, which are characterized by the Koszul complex. Now let R, I be as above but A a finitely generated R-module. The notions of A-sequence and of $\operatorname{depth}_I A$ are well-known in commutative algebra. $\operatorname{Depth}_I A$ is characterized by the Koszul complex or alternatively using the functor Ext.

We see a parallelism in these notions; we find correspondence between A-sequence and A-cosequence and between $\operatorname{depth}_I A$ and I-dimension of A. For this reason, we shall call the latter the *I-codepth* of A and write $\operatorname{codepth}_I A$. We shall show in §1 of this paper that it can be characterized by Ext. We shall show some more properties of codepth in §2, and give some examples in §3.

Throughout this paper, R is a commutative Noetherian ring with 1. If A is an R-module, then $E(A)$ denotes the injective envelope of A. If I is an ideal of R then $V(I)$ denotes the set of prime ideals containing I.

§1. Characterization of codepth by Ext.

Definition. Let R be a Noetherian ring, I an ideal of R, A an Artinian R-module and $x_1, \cdots, x_n$ elements of R. Then a sequence $x_1, \cdots, x_n$ is said to be an *A-cosequence* if

1) $E_i \xrightarrow{x_{i+1}} E_i \longrightarrow 0$ exact $(i=0, 1, \cdots, n-1)$
where $E_0=A$, $E_i=0:_A(x_1, \cdots, x_i)$ if $i\neq 0$.
2) $E_n=0:_A(x_1, \cdots, x_n)\neq 0$.

Remark. Let R, I, A be as above. If $x_1, \cdots, x_n$ is an A-cosequence in I, then the ideals (x_1), (x_1, x_2), $\cdots$, $(x_1, x_2, \cdots, x_n)$ form a properly ascending chain. Therefore, every A-cosequence can be extended to a maximal one which has finite length.

Definition. Let R be a Noetherian ring with a proper ideal I. Let A be an Artinian R-module. Then the *I-codepth* of A, $\operatorname{codepth}_I A$ is the length of the longest A-cosequence in I.

If R is a local ring with a maximal ideal M, $\operatorname{codepth}_M A$ is called simply the codepth of A, codepth A.

Theorem 1. *Let A be an Artinian R-module, I an ideal of R with $0:_A I\neq 0$ and E an injective cogenerator of R. A^* will denote* $\operatorname{Hom}_R(A, E)$. *Let $n>0$ be an integer, then the following statements are equivalent.*