44. On the Kazhdan-Lusztig Conjecture for Kac-Moody Algebras

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(Communicated by Shokichi IYANAGA, M. J. A., April 14, 1986)

Let $\mathfrak{g}$ be a Kac-Moody algebra over $\boldsymbol{C}$ and $\mathfrak{h}$ a Cartan subalgebra of $\mathfrak{g}$. Let $\rho \in \mathfrak{h}^*$ be an element which takes the value 1 on each simple coroot and W the Weyl group of $\mathfrak{g}$. We denote by $M(\lambda)$ and $L(\lambda)$ the Verma module with highest weight $\lambda \in \mathfrak{h}^*$ and the unique irreducible quotient of $M(\lambda)$ respectively.

If the generalized Cartan matrix (GCM) corresponding to $\mathfrak{g}$ is symmetrizable, then it was proved in [3] analogously to the (finite-dimensional) complex semisimple case that for any dominant integral element $\lambda \in \mathfrak{h}^*$ and $y \in W$, all the irreducible subquotients of $M(y(\lambda+\rho)-\rho)$ are $L(w(\lambda+\rho)-\rho)$ with $w \in W$ such that $w \geqq y$, and that multiplicities $\operatorname{mtp}(y, w)=[M(y(\lambda+\rho)-\rho) : L(w(\lambda+\rho)-\rho)]$ are independent of λ. Here $\leqq$ is the standard partial order on the Coxeter group W in which the unit is the smallest element. Note that $\operatorname{mtp}(y, w)=0$ if $y \not\leqq w$.

In case where $\mathfrak{g}$ is finite-dimensional, the following proposition on these multiplicities, well-known as the Kazhdan-Lusztig conjecture, was proved in [2] and in [1] independently.

Theorem A [5, Conjecture 1.5]. *Let $\mathfrak{g}$ be a complex semisimple Lie algebra. Under the same notations as above,*

$$\operatorname{mtp}(y, w)=P_{y,w}(1)$$

holds for all $y, w \in W$ such that $y \leqq w$, where $P_{y,w}$ are the Kazhdan-Lusztig polynomials for the Coxeter group W.

Kazhdan-Lusztig polynomials were introduced in [5], related to a base change of Hecke algebras of Coxeter groups, and there were given inductive formulas to compute these polynomials.

Deodhar, Gabber and Kac conjectured in [3] that the same result as Theorem A holds for Kac-Moody algebras of infinite-dimension as follows.

Conjecture B [3, Conjecture 5.16]. Let $\mathfrak{g}$ be a Kac-Moody algebra corresponding to a symmetrizable GCM. Then

$$\operatorname{mtp}(y, w)=P_{y,w}(1)$$

holds for all $y, w \in W$ such that $y \leqq w$.

In this paper, we prove this conjecture is true for certain pairs (y, w), by reducing it to the finite-dimensional case. Even when the GCM is not symmetrizable, this holds for $\lambda=0$, the most important case. We give further a branching rule of Verma modules over a non-twisted affine Lie algebra with respect to a certain subalgebra.