38. An Average Effect of Many Tiny Holes in Nonlinear Boundary Value Problems with Monotone Boundary Conditions

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1. Introduction. Recently H. Attouch has showed in his treatise [1] that it is possible to compute a magnitude effect of many tiny holes, of which shapes are not spherical in general, to the potential term of the Dirichlet problem for the Laplacian, using the notion of capacity. Here we show that the method in [1] can be extended to a different type of boundary conditions, nonlinear boundary conditions, by replacing capacity by a different class of magnitude. This paper is a generalization and an improvement of the work [5].

Let $F(\ni 0)$ be a closed subset of $\boldsymbol{R}^N$, $N\geqq 3$, with non-empty interior and its diameter 2. Let $\boldsymbol{R}^N$ be divided into cubes C_ε^i of volume ε^N and x_ε^i the center of C_ε^i, $i\in \boldsymbol{N}$. We set $F_\varepsilon^i=x_e^i+r_\varepsilon F$ with small $r_\varepsilon>0$. Let Ω be a bounded domain of $\boldsymbol{R}^N$ with smooth boundary Γ. From Ω we remove all holes F_ε^i such that dist $(\Gamma, F_\varepsilon^i)\geqq\varepsilon$ and obtain Ω_ε. We assume that the complement $\boldsymbol{c}F$ of F consists of an unbounded component and has a smooth boundary. We consider the following monotone boundary value problem (cf. [3]) : for $f\in L^2(\Omega)$ find $u_\varepsilon\in H^1(\Omega_\varepsilon)$ such that

$$(1)\qquad \begin{cases} -\Delta u_\varepsilon=f & \text{a.e. in } \Omega_\varepsilon, \\ \dfrac{\partial u_\varepsilon}{\partial\nu}+\beta_\varepsilon(u_\varepsilon)=0 & \text{a.e. on } \partial\Omega_\varepsilon, \end{cases}$$

where $\partial/\partial\nu$ denotes the outward normal derivative on $\partial\Omega_\varepsilon$ and β_ε is a function : $\boldsymbol{R}\to\boldsymbol{R}$ defined by (i) $\beta_\varepsilon(r)=(r+c_\varepsilon)/L_\varepsilon$, $r\leqq -c_\varepsilon$, (ii) $\beta_\varepsilon(r)=0$, $|r|\leqq c_\varepsilon$, (iii) $\beta_\varepsilon(r)=(r-c_\varepsilon)/L_\varepsilon$, $r\geqq c_\varepsilon$. The problem (1) admits a unique solution $u_\varepsilon\in H^2(\Omega_\varepsilon)$ (cf. also [3]). We consider the behavior of u_ε under the condition

$$(2)\qquad \sup L_\varepsilon<\infty,\quad c_\varepsilon\to 0,\quad r_\varepsilon\to 0 \quad\text{and}\quad n_\varepsilon\to\infty,$$

where n_ε denotes the number of holes of Ω_ε.

We introduce a class of magnitude on a closed set F, determined by the shape of ∂F and a sequence $\{r_\varepsilon, \beta_\varepsilon\}_\varepsilon$ by

$$(3)\qquad C_{\partial F}=\lim_{\substack{R\to\infty\\ r_\varepsilon\to 0}}\gamma(R, r_\varepsilon),$$

$$(4)\qquad \gamma(R, r_\varepsilon)=\inf\left\{\int_{B_R\backslash F}|\nabla v|^2dx+r_\varepsilon\int_{\partial F}v\beta_\varepsilon(v)d\sigma : v\in W_R\right\}$$

where $B_R=\{x\in\boldsymbol{R}^N : |x|<R\}$ and $W_R=\{v\in H^1(B_R\backslash F) : v\geqq 1 \text{ on } \partial B_R\}$. We can show that the value $C_{\partial F}$ is well defined under the condition

$$(5)\qquad L_\varepsilon/r_\varepsilon\to q \qquad\text{as}\quad r_\varepsilon\to 0,$$