# 25. The $L^p$-boundedness of Pseudo-differential Operators Satisfying Estimates of Parabolic Type and Product Type. II

By Masao YAMAZAKI
Department of Mathematics, University of Tokyo

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We stated in our previous paper (Yamazaki [6]) the $L^p$-boundedness of pseudo-differential operators with non-smooth symbols satisfying non-classical estimates. A proof will be given in the forthcoming paper (Yamazaki [7]).

On the other hand, Bourdaud [1] and Nagase [4] generalized the $L^p$-boundedness theorem of Coifman-Meyer [2] and Muramatu-Nagase [3] on the classical symbols, by considering the combined effect of the $x$-regularity and the $\xi$-growth of the symbols.

Here we consider a similar effect where the symbols satisfy non-classical estimates. Our main theorem is an improvement of Theorem 4 of [7].

**1. Notations and definitions.** Let $n(1), \cdots, n(N)$ be positive integers. We put $n=n(1)+\cdots+n(N)$ and

$$\Lambda(\nu)=\{l \in \boldsymbol{N};\ n(1)+\cdots+n(\nu-1)+1 \leqq l \leqq n(1)+\cdots+n(\nu)\}$$

for $\nu=1, \cdots, n$.

We regard $\boldsymbol{R}^n$ as $\boldsymbol{R}^{n(1)}\times\cdots\times\boldsymbol{R}^{n(N)}$, and write $x \in \boldsymbol{N}^n$ as $x=(x^{(1)}, \cdots, x^{(N)})$, where $x^{(\nu)}=(x_l)_{l\in\Lambda(\nu)}$. We also give a weight $M=(M^{(1)}, \cdots, M^{(N)})$ to the coordinate variables of $\boldsymbol{R}^n$, where each $M^{(\nu)}=(m_l)_{l\in\Lambda(\nu)}$ satisfies the condition $\min_{l\in\Lambda(\nu)} m_l=1$.

Next, for every $\nu=1, \cdots, N$, we define a function $[y]_\nu$ of $y=(y_l)_{l\in\Lambda(\nu)} \in \boldsymbol{R}^{n(\nu)}$ with values in $\boldsymbol{R}^+=\{t\,;\,t\geqq 0\}$ as follows. We put $[0]_\nu=0$, and if $y\neq 0$, let $[y]_\nu$ denote the unique positive root of the equation $\sum_{l\in\Lambda(\nu)} t^{-2m_l}y_l^2=1$ with respect to $t$.

Further, for $\nu=1, 2, \cdots, N$ and $y \in \boldsymbol{R}^{n(\nu)}$, let $\Delta_y^{(\nu)}$ denote the difference of the first order with respect to the $\nu$-th part of the coordinate variables; that is, we put

$$\Delta_y^{(\nu)} f(x)=f(x^{(1)}, \cdots, x^{(\nu)}-y, \cdots, x^{(N)})-f(x)$$

for a function $f(x)$ on $\boldsymbol{R}^n$. We also fix a positive number $L$.

Now we introduce a notion to state our main theorem.

**Definition.** We call a family of functions $\{\omega_1(s_1\,;t_1), \omega_2(s_1, s_2\,;t_1, t_2), \cdots, \omega_N(s_1, s_2, \cdots, s_N\,;t_1, t_2, \cdots, t_N)\}$ a *multiple modulus of growth and continuity* if it satisfies the following four conditions:

1) For every $\nu$, the function $\omega_\nu(s_1, \cdots, s_\nu\,;t_1, \cdots, t_\nu)$ is a function on $(\boldsymbol{R}^+)^{2\nu}$ into $\boldsymbol{R}^+$, and is monotone-increasing and concave with respect to each $t_k$.